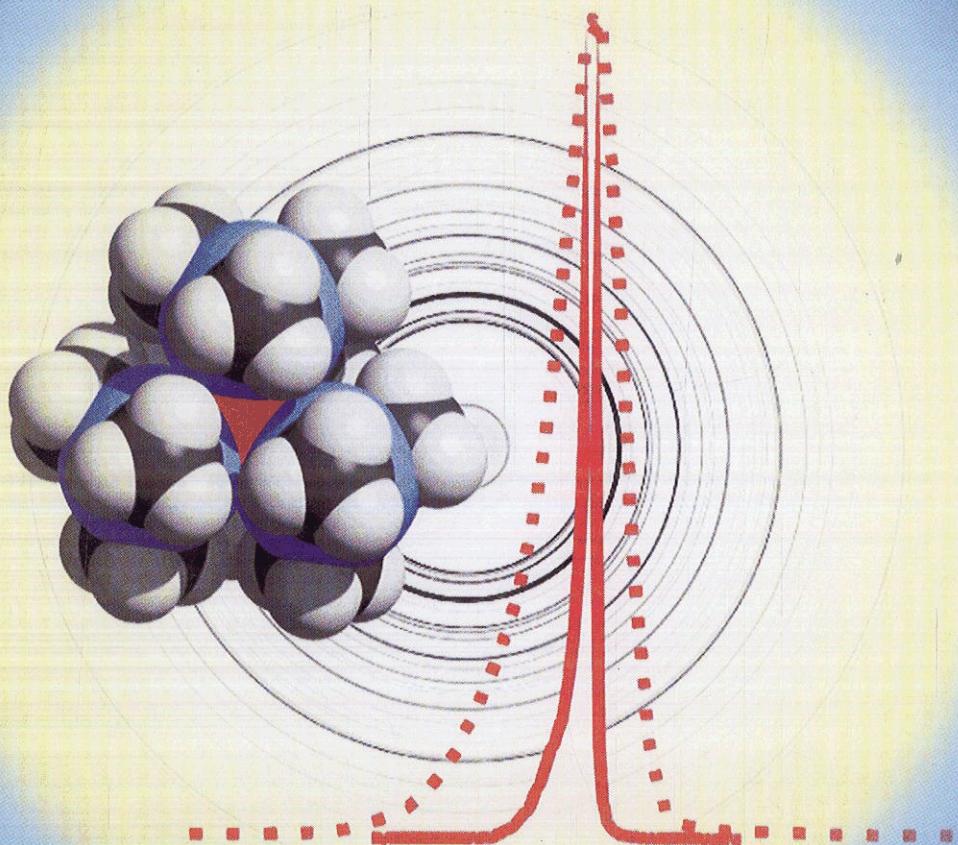


# Powder Diffraction

## Theory and Practice

Edited by R E Dinnebier and S J L Billinge



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