

Wiley Series in Special Topics in Inorganic Chemistry • R. Bruce King, Series Editor

# Defects in SOLIDS

Richard J. D. Tilley

# CONTENTS

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|   |           |
|---|-----------|
| <b>Preface</b>  | <b>xv</b> |
| <b>1. Point Defects</b>   | <b>1</b>  |
| 1.1 Introduction  | 1         |
| 1.2 Point and Electronic Defects in Crystalline Solids                  | 4         |
| 1.3 Electronic Properties: Doped Silicon and Germanium as Examples      | 7         |
| 1.4 Optical Properties: F Centers and Ruby as Examples                  | 10        |
| 1.5 Bulk Properties   | 12        |
| 1.5.1 Unit Cell Dimensions  | 13        |
| 1.5.2 Density   | 14        |
| 1.5.3 Volume  | 16        |
| 1.5.4 Young's Modulus (the Elastic Modulus)                             | 17        |
| 1.6 Thermoelectric Properties: The Seebeck Coefficient as an Example    | 18        |
| 1.7 Point Defect Notation   | 20        |
| 1.8 Charges on Defects  | 21        |
| 1.8.1 Electrons and Electron Holes                                      | 21        |
| 1.8.2 Atomic and Ionic Defects  | 22        |
| 1.9 Balanced Populations of Point Defects: Schottky and Frenkel Defects | 25        |
| 1.9.1 Schottky Defects  | 25        |
| 1.9.2 Frenkel Defects   | 27        |
| 1.10 Antisite Defects   | 28        |
| 1.11 Defect Formation and Reaction Equations                            | 31        |
| 1.11.1 Addition and Subtraction of Atom                                 | 31        |
| 1.11.2 Equation Formalism   | 31        |
| 1.11.3 Formation of Antisite Defects                                    | 33        |
| 1.11.4 Nickel Oxide   | 33        |
| 1.11.5 Cadmium Oxide  | 35        |
| 1.11.6 Calcia-stabilized Zirconia                                       | 36        |
| 1.11.7 Ternary Oxides   | 37        |
| 1.12 Combinations of Point Defects in Pure Materials                    | 38        |
| 1.13 Structural Consequences of Point Defect Populations                | 39        |
| 1.14 Answers to Introductory Questions                                  | 40        |

|   |           |
|---|-----------|
| Problems and Exercises  | 41        |
| References  | 44        |
| Further Reading   | 44        |
| <b>2. Intrinsic Point Defects in Stoichiometric Compounds</b>           | <b>45</b> |
| 2.1 Equilibrium Population of Vacancies in a Monatomic Crystal          | 45        |
| 2.2 Equilibrium Population of Self-Interstitials in a Monatomic Crystal | 50        |
| 2.3 Equilibrium Population of Schottky Defects in a Crystal             | 52        |
| 2.4 Lithium Iodide Battery  | 54        |
| 2.5 Equilibrium Population of Frenkel Defects in a Crystal              | 56        |
| 2.6 Photographic Film   | 58        |
| 2.7 Photochromic Glasses  | 62        |
| 2.8 Equilibrium Population of Antisite Defects in a Crystal             | 64        |
| 2.9 Intrinsic Defects: Trends and Further Considerations                | 65        |
| 2.10 Computation of Defect Energies                                     | 66        |
| 2.10.1 Defect Calculations  | 66        |
| 2.10.2 Point Defect Interactions  | 68        |
| 2.10.3 Atomistic Simulation   | 70        |
| 2.10.4 The Shell Model  | 72        |
| 2.10.5 Defect Formation Energy  | 75        |
| 2.10.6 Quantum Mechanical Calculations                                  | 76        |
| 2.11 Answers to Introductory Questions                                  | 77        |
| Problems and Exercises  | 78        |
| References  | 81        |
| Further Reading   | 81        |
| <b>3. Extended Defects</b>  | <b>83</b> |
| 3.1 Dislocations  | 83        |
| 3.2 Edge Dislocations   | 85        |
| 3.3 Screw Dislocations  | 90        |
| 3.4 Mixed Dislocations  | 93        |
| 3.5 Unit and Partial Dislocations                                       | 94        |
| 3.6 Multiplication of Dislocations                                      | 99        |
| 3.7 Interaction of Dislocations and Point Defects                       | 99        |
| 3.7.1 Dislocation Loops   | 99        |
| 3.7.2 Dislocation Climb   | 102       |
| 3.7.3 Decoration of Dislocations  | 103       |
| 3.8 Dislocations in Nonmetallic Crystals                                | 103       |
| 3.9 Internal Boundaries   | 107       |
| 3.10 Low-Angle Grain Boundaries   | 108       |
| 3.11 Twin Boundaries  | 110       |
| 3.12 Antiphase Boundaries   | 114       |

|   |            |
|---|------------|
| 3.13 Domains and Ferroic Materials  | 116        |
| 3.13.1 Magnetic Structures  | 116        |
| 3.13.2 Ferroelectric Structures   | 118        |
| 3.13.3 Ferroic Structures   | 119        |
| 3.14 External Surfaces and Grain Boundaries   | 120        |
| 3.14.1 Optical Characteristics of Polycrystalline Solids  | 122        |
| 3.14.2 Electronic Properties of Interfaces  | 122        |
| 3.14.3 Varistors  | 124        |
| 3.14.4 Positive Temperature Coefficient Thermistors   | 126        |
| 3.15 Volume Defects and Precipitates  | 128        |
| 3.16 Answers to Introductory Questions  | 129        |
| Problems and Exercises  | 131        |
| Further Reading   | 133        |
| <b>4. Structural Aspects of Composition Variation</b>   | <b>135</b> |
| 4.1 Composition Variation and Nonstoichiometry  | 135        |
| 4.1.1 Phase Diagrams and the Coexistence of Solids  | 135        |
| 4.1.2 Nonstoichiometric Compounds   | 137        |
| 4.1.3 Phase Diagrams and Composition  | 137        |
| 4.2 Substitutional Solid Solutions  | 140        |
| 4.3 Point Defects and Departures from Stoichiometry   | 143        |
| 4.3.1 Substitution: Gallium Arsenide, GaAs  | 143        |
| 4.3.2 Vacancies: Cobalt Oxide, CoO  | 145        |
| 4.3.3 Interstitials: La <sub>2</sub> CuO <sub>4</sub> and Sr <sub>2</sub> CuO <sub>2</sub> F <sub>2</sub> | 145        |
| 4.3.4 Interstitial Impurities: Alloys and Hydrides  | 147        |
| 4.3.5 Defect Variation: Zinc Oxide, ZnO   | 147        |
| 4.3.6 Defect Spinels, M <sub>2</sub> O <sub>3</sub>   | 148        |
| 4.4 Defect Clusters   | 149        |
| 4.4.1 Point Defect Aggregations   | 149        |
| 4.4.2 Iron Oxide, Wüstite   | 150        |
| 4.4.3 Hyperstoichiometric Uranium Dioxide   | 152        |
| 4.4.4 Anion-Excess Fluorite Structures  | 155        |
| 4.4.5 Anion-Deficient Fluorite Structures   | 157        |
| 4.5 Interpolation   | 161        |
| 4.6 Intercalation   | 165        |
| 4.7 Linear Defects  | 167        |
| 4.8 Modular Structures  | 170        |
| 4.8.1 Crystallographic Shear Phases   | 171        |
| 4.8.2 Twinning and Composition Variation  | 176        |
| 4.8.3 Chemical Twinning   | 177        |
| 4.8.4 Perovskite-Related Structures   | 179        |
| 4.8.5 Misfit-Layer Compounds  | 184        |
| 4.8.6 Niobium Pentoxide Block Structures  | 187        |

|           |  |            |
|-----------|--|------------|
| 4.9       | Ordering and Assimilation  | 190        |
| 4.10      | Modulated Structures   | 192        |
| 4.10.1    | Structure Modulations  | 192        |
| 4.10.2    | Yttrium Oxyfluoride and Related Structures                               | 193        |
| 4.10.3    | $U_3O_8$ -Related Structures   | 195        |
| 4.10.4    | $Sr_xTiS_3$ and Related Structures                                       | 196        |
| 4.11      | Answers to Introductory Questions  | 198        |
|           | Problems and Exercises   | 199        |
|           | Further Reading  | 202        |
| <b>5.</b> | <b>Defects and Diffusion</b>   | <b>205</b> |
| 5.1       | Diffusion  | 205        |
| 5.2       | Diffusion in Solids  | 207        |
| 5.3       | Random-Walk Diffusion in Crystals  | 213        |
| 5.4       | Diffusion Mechanisms   | 216        |
| 5.4.1     | Vacancy Diffusion  | 216        |
| 5.4.2     | Interstitial Diffusion   | 217        |
| 5.4.3     | Interstitialcy Diffusion   | 217        |
| 5.4.4     | Impurity and Cluster Diffusion Mechanisms                                | 219        |
| 5.4.5     | Diffusion Paths  | 221        |
| 5.5       | Point Defect Concentration and Diffusion                                 | 227        |
| 5.6       | Correlation Factors  | 227        |
| 5.7       | Temperature Variation of the Diffusion Coefficient                       | 231        |
| 5.8       | Temperature Variation and Intrinsic Diffusion                            | 236        |
| 5.9       | Diffusion Mechanisms and Impurities                                      | 240        |
| 5.10      | Chemical and Ambipolar Diffusion   | 241        |
| 5.11      | Dislocation and Grain Boundary Diffusion                                 | 243        |
| 5.12      | Diffusion in Amorphous and Glassy Solids                                 | 245        |
| 5.13      | Answers to Introductory Questions  | 245        |
|           | Problems and Exercises   | 246        |
|           | Further Reading  | 250        |
| <b>6.</b> | <b>Intrinsic and Extrinsic Defects in Insulators: Ionic Conductivity</b> | <b>251</b> |
| 6.1       | Ionic Conductivity   | 251        |
| 6.1.1     | Ionic Conductivity in Solids   | 251        |
| 6.1.2     | Fundamental Concepts   | 252        |
| 6.2       | Mechanisms of Ionic Conductivity   | 255        |
| 6.2.1     | Random-Walk Model  | 255        |
| 6.2.2     | Relationship between Ionic Conductivity and Diffusion Coefficient        | 259        |
| 6.2.3     | Ionic Conductivity and Defects   | 261        |
| 6.3       | Impedance Measurements   | 263        |
| 6.4       | Electrochemical Cells and Batteries                                      | 265        |

|           |   |            |
|-----------|---|------------|
| 6.5       | Disordered Cation Compounds                                       | 268        |
| 6.6       | $\beta$ -Alumina Oxides   | 270        |
| 6.6.1     | Idealized Structures of the $\beta$ -Alumina-Related Phases       | 270        |
| 6.6.2     | Defects in $\beta$ -Alumina                                       | 272        |
| 6.6.3     | Defects in $\beta''$ -Alumina                                     | 274        |
| 6.6.4     | Ionic Conductivity  | 274        |
| 6.6.5     | Batteries Using $\beta''$ -Alumina                                | 276        |
| 6.7       | Enhancement of Ionic Conductivity                                 | 278        |
| 6.8       | Calcia-Stabilized Zirconia and Related Fast Oxygen Ion Conductors | 280        |
| 6.8.1     | Structure and Oxygen Diffusion in Fluorite Structure Oxides       | 280        |
| 6.8.2     | Free Energy and Stoichiometry of Oxides                           | 281        |
| 6.8.3     | Oxygen Sensors  | 282        |
| 6.8.4     | Oxygen Pumps and Coulometric Titrations                           | 284        |
| 6.9       | Proton ( $H^+$ Ion) Conductors                                    | 285        |
| 6.10      | Solid Oxide Fuel Cells  | 287        |
| 6.11      | Answers to Introductory Questions                                 | 288        |
|           | Problems and Exercises  | 290        |
|           | Further Reading   | 294        |
| <b>7.</b> | <b>Nonstoichiometry and Intrinsic Electronic Conductivity</b>     | <b>297</b> |
| 7.1       | Nonstoichiometry and Electronic Defects in Oxides                 | 297        |
| 7.1.1     | Electronic and Ionic Compensation                                 | 297        |
| 7.1.2     | Metal-Excess Phases   | 298        |
| 7.1.3     | Oxygen-Excess Phases  | 299        |
| 7.2       | Conductivity and Defects  | 301        |
| 7.2.1     | Conductivity and Defect Concentrations                            | 301        |
| 7.2.2     | Holes, Electrons, and Valence                                     | 302        |
| 7.2.3     | Localized Electrons and Polarons                                  | 303        |
| 7.2.4     | Defects and Hopping Conductivity                                  | 304        |
| 7.2.5     | Band versus Hopping Conduction                                    | 306        |
| 7.2.6     | Seebeck Coefficient and Stoichiometry                             | 307        |
| 7.3       | Stoichiometry, Defect Populations and Partial Pressures           | 309        |
| 7.3.1     | Equilibrium Partial Pressures                                     | 309        |
| 7.3.2     | Phase Rule  | 312        |
| 7.4       | Variation of Defect Populations with Partial Pressure             | 315        |
| 7.4.1     | Metal-Excess Oxides   | 315        |
| 7.4.2     | Oxygen-Excess Oxides  | 317        |
| 7.4.3     | $Ba_2In_2O_5$   | 318        |
| 7.5       | Brouwer Diagrams  | 320        |
| 7.5.1     | Initial Assumptions   | 320        |
| 7.5.2     | Defect Equilibria   | 321        |
| 7.5.3     | Stoichiometric Point: Ionic Defects                               | 322        |

|       |  |     |
|-------|--|-----|
| 7.5.4 | Near Stoichiometry: Ionic Defects                | 324 |
| 7.5.5 | High $X_2$ Partial Pressures: Ionic Defects      | 326 |
| 7.5.6 | Low $X_2$ Partial Pressures: Ionic Defects       | 327 |
| 7.5.7 | Complete Diagram: Ionic Defects                  | 328 |
| 7.6   | Brouwer Diagrams: Electronic Defects             | 329 |
| 7.6.1 | Electronic Defects                               | 329 |
| 7.6.2 | Near Stoichiometry: Electronic Defects           | 330 |
| 7.6.3 | High $X_2$ Partial Pressures: Electronic Defects | 333 |
| 7.6.4 | Low $X_2$ Partial Pressures: Electronic Defects  | 334 |
| 7.6.5 | Complete Diagram: Electronic Defects             | 335 |
| 7.7   | Brouwer Diagrams: More Complex Examples          | 336 |
| 7.7.1 | $\text{Cr}_2\text{O}_3$                          | 336 |
| 7.7.2 | $\text{Ba}_2\text{In}_2\text{O}_5$               | 338 |
| 7.8   | Brouwer Diagrams: Effects of Temperature         | 339 |
| 7.9   | Polynomial Forms for Brouwer Diagrams            | 340 |
| 7.9.1 | Ionic Defects                                    | 340 |
| 7.9.2 | Electronic Defects                               | 343 |
| 7.10  | Answers to Introductory Questions                | 344 |
|       | Problems and Exercises                           | 345 |
|       | References                                       | 349 |
|       | Further Reading                                  | 349 |

|           |   |            |
|-----------|---|------------|
| <b>8.</b> | <b>Nonstoichiometry and Extrinsic Electronic Conductivity</b> | <b>351</b> |
| 8.1       | Effect of Impurity Atoms                                      | 351        |
| 8.2       | Impurities in Oxides  | 352        |
| 8.2.1     | Donor Doping  | 352        |
| 8.2.2     | Donor Doping of $\text{Cr}_2\text{O}_3$                       | 352        |
| 8.2.3     | Acceptor Doping   | 353        |
| 8.2.4     | Acceptor Doping of $\text{NiO}$                               | 354        |
| 8.3       | Negative Temperature Coefficient (NTC) Thermistors            | 355        |
| 8.4       | Brouwer Diagrams for Doped Systems                            | 358        |
| 8.4.1     | Construction  | 358        |
| 8.4.2     | General Trends: MX  | 360        |
| 8.5       | Metals and Insulators   | 363        |
| 8.5.1     | Acceptor Doping into $\text{La}_2\text{CuO}_4$                | 363        |
| 8.5.2     | Donor Doping into $\text{Nd}_2\text{CuO}_4$                   | 365        |
| 8.5.3     | Spinel System $\text{Li}_{1+x}\text{Ti}_{2-x}\text{O}_4$      | 366        |
| 8.6       | Cuprate High-Temperature Superconductors                      | 367        |
| 8.6.1     | Perovskite-Related Structures and Series                      | 367        |
| 8.6.2     | Hole Doping   | 370        |
| 8.6.3     | Defect Structures   | 373        |

|           |   |            |
|-----------|---|------------|
| 8.7       | Mixed Electronic/Ionic Conductors                           | 378        |
| 8.7.1     | Fluorite Structure Oxides: $\text{CeO}_{2-\delta}$          | 378        |
| 8.7.2     | Layered Structures: $\text{Li}_x\text{MX}_2$                | 380        |
| 8.7.3     | Acceptor Doping in Perovskite Structure Oxides              | 381        |
| 8.8       | Mixed Proton/Electronic Conductors                          | 385        |
| 8.8.1     | Proton Mixed Conductors                                     | 385        |
| 8.8.2     | Brouwer Diagram Representation of Mixed Proton Conductivity | 387        |
| 8.8.3     | Charge Carrier Map Representation of Mixed Conductivity     | 389        |
| 8.9       | Choice of Compensation Mechanism                            | 391        |
| 8.10      | Answers to Introductory Questions                           | 392        |
|           | Problems and Exercises                                      | 394        |
|           | Further Reading   | 397        |
| <b>9.</b> | <b>Magnetic and Optical Defects</b>                         | <b>399</b> |
| 9.1       | Magnetic Defects  | 399        |
| 9.2       | Magnetic Defects in Semiconductors                          | 403        |
| 9.3       | Magnetic Defects in Ferrites                                | 405        |
| 9.4       | Charge and Spin States in Cobaltites and Manganites         | 406        |
| 9.5       | Extended Magnetic Defects                                   | 409        |
| 9.6       | Optical Defects   | 412        |
| 9.6.1     | Absorption and Scattering                                   | 412        |
| 9.6.2     | Energy Levels   | 413        |
| 9.6.3     | Energy Levels in Solids                                     | 416        |
| 9.7       | Pigments, Minerals and Gemstones                            | 417        |
| 9.7.1     | Transition-Metal and Lanthanide Ion Colors                  | 417        |
| 9.7.2     | Colors and Impurity Defects                                 | 418        |
| 9.8       | Photoluminescence   | 420        |
| 9.8.1     | Energy Degradation and Down-conversion                      | 420        |
| 9.8.2     | Up-conversion   | 421        |
| 9.9       | Solid-State Lasers  | 428        |
| 9.9.1     | Emission of Radiation                                       | 428        |
| 9.9.2     | Ruby Laser: Three-Level Lasers                              | 429        |
| 9.9.3     | Neodymium ( $\text{Nd}^{3+}$ ) Laser: Four-Level Lasers     | 430        |
| 9.10      | Color Centers   | 432        |
| 9.10.1    | The F Center  | 432        |
| 9.10.2    | Electron and Hole Centers                                   | 433        |
| 9.10.3    | Surface Color Centers                                       | 435        |
| 9.10.4    | Complex Color Centers: Laser and Photonic Action            | 436        |
| 9.11      | Electrochromic Films  | 438        |
| 9.12      | Photoinduced Magnetism                                      | 440        |
| 9.13      | Answers to Introductory Questions                           | 441        |

|   |            |
|---|------------|
| Problems and Exercises  | 443        |
| Further Reading   | 446        |
| <b>Supplementary Material</b>   | <b>447</b> |
| S1 Crystal Structures   | 447        |
| S1.1 Crystal Systems and Unit Cells   | 447        |
| S1.2 Crystal Planes and Miller Indices  | 448        |
| S1.3 Directions   | 451        |
| S1.4 Crystal Structures   | 452        |
| Further Reading   | 460        |
| S2 Band Theory  | 461        |
| S2.1 Energy Bands   | 461        |
| S2.2 Insulators, Semiconductors and Metals  | 461        |
| S2.3 Point Defects and Energy Bands in Semiconductors and Insulators                    | 463        |
| S2.4 Transition-Metal Oxides  | 464        |
| S3 Seebeck Coefficient  | 465        |
| S3.1 Seebeck Coefficient and Entropy  | 465        |
| S3.2 Seebeck Coefficient and Defect Populations   | 468        |
| S4 Schottky and Frenkel Defects   | 470        |
| S4.1 Equilibrium Concentration of Schottky Defects Derived from Configurational Entropy | 470        |
| S4.2 Stirling's Approximation   | 473        |
| S4.3 Equilibrium Concentration of Frenkel Defects Derived from Configurational Entropy  | 474        |
| S5 Diffusion  | 476        |
| S5.1 Diffusion Equations  | 476        |
| S5.2 Non-Steady-State Diffusion   | 476        |
| S5.3 Random-Walk Diffusion  | 478        |
| S5.4 Concentration Profile  | 484        |
| S5.5 Fick's Laws and the Diffusion Equations  | 485        |
| S5.6 Penetration Depth  | 489        |
| S6 Magnetic Properties  | 489        |
| S6.1 Atomic Magnetism   | 489        |
| S6.2 Types of Magnetic Material   | 490        |
| S6.3 Crystal Field Splitting  | 492        |
| <b>Answers to Problems and Exercises</b>  | <b>495</b> |
| <b>Formula Index</b>  | <b>507</b> |
| <b>Subject Index</b>  | <b>513</b> |