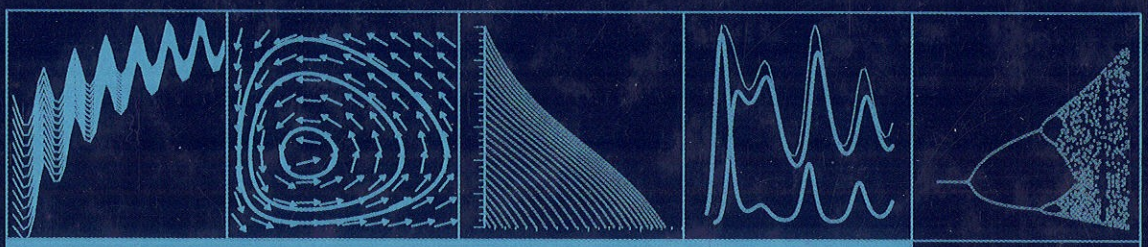
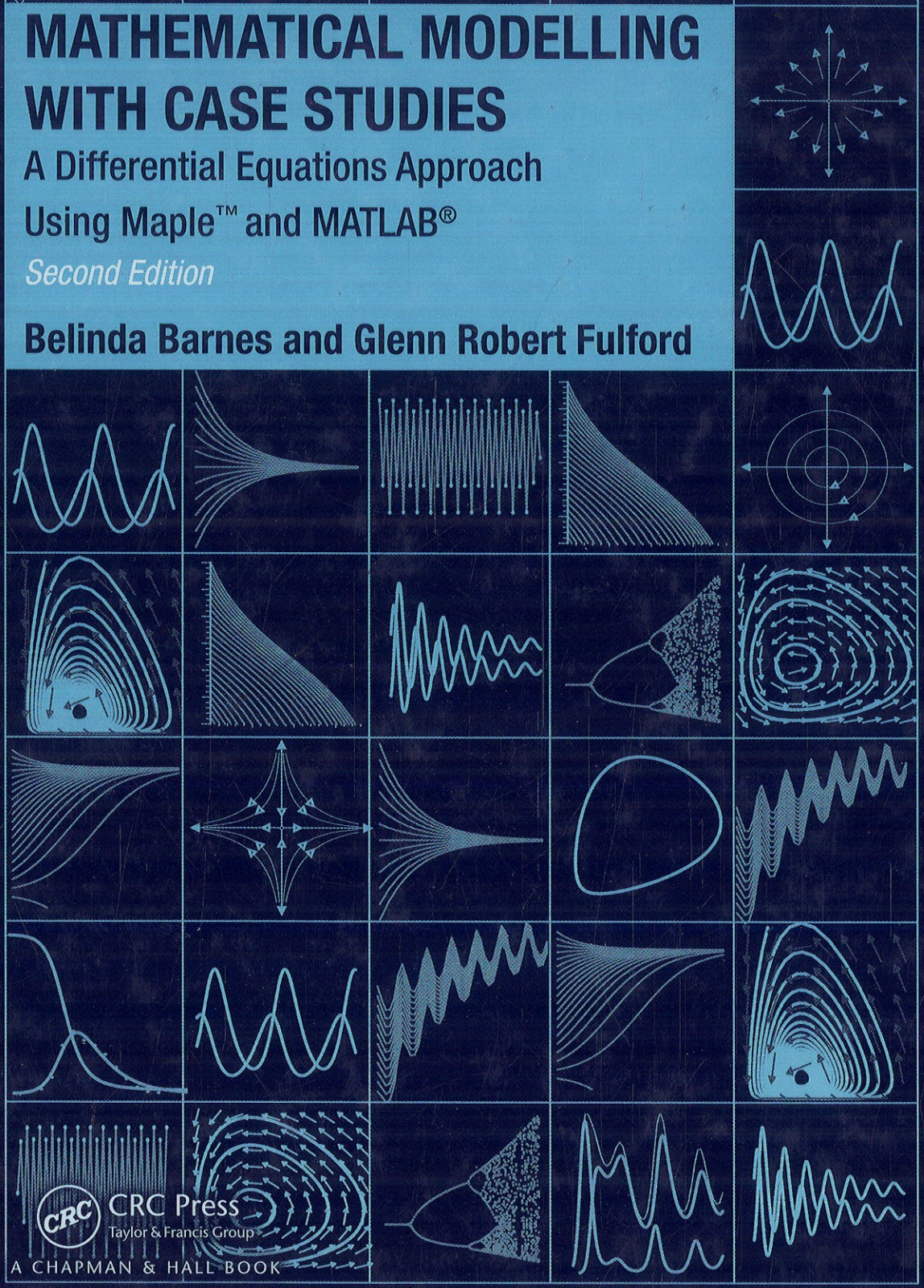


MATHEMATICAL MODELLING WITH CASE STUDIES

A Differential Equations Approach Using Maple™ and MATLAB®

Second Edition

Belinda Barnes and Glenn Robert Fulford



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