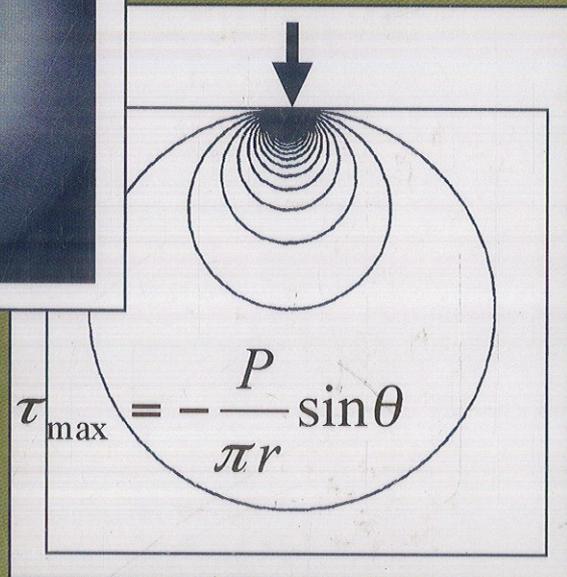
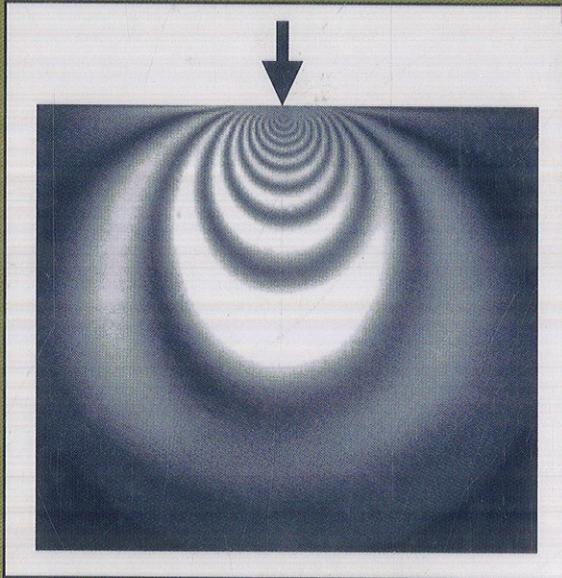


2e

Elasticity

Theory, Applications, and Numerics

Martin H. Sadd



$$\nabla^4 \phi = 0 \Rightarrow$$



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