

Real Infinite Series

$$1 + \frac{2}{3} + \frac{2}{5} + \frac{2}{9} + \frac{2}{17} + \dots = \sum_{n=0}^{\infty} \frac{2}{2^n + 1}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) = 1$$

$$\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}} \geq \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

$$\frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \dots = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}} \geq \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$$

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