

Foundations and
Applications of Mechanics

Volume I

Continuum Mechanics

Second Edition

C.S. JOG



Alpha
Science

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