

Foundations and
Applications of Mechanics

Volume II

Fluid Mechanics

Second Edition

C.S. JOG



Alpha
Science

Contents

Preface	v
Notation	vii
1 Kinematics and Governing Equations	1
1.1 Tensor Identities	1
1.2 Kinematics	2
1.2.1 Lagrangian and Eulerian descriptions	2
1.2.2 Flow lines	5
1.2.3 Analysis of deformation	6
1.2.4 Vortex lines and circulation	11
1.3 Governing Equations	12
1.3.1 Conservation of mass	14
1.3.2 Balance of linear momentum	15
1.3.3 Balance of angular momentum	17
1.3.4 Principle of material frame-indifference	18
1.3.5 Definition of a fluid	25
1.3.6 The first law of thermodynamics	26
1.3.7 The second law of thermodynamics	29
1.3.8 The Navier-Stokes and energy equations	40
1.3.9 Summary of the governing equations for a Newtonian fluid	41
1.4 Nature of the Governing Equations	42
1.5 Boundary Conditions on Velocity and Temperature	43
1.5.1 Kinematic boundary conditions on the velocity field	43
1.5.2 Dynamical boundary conditions at a fluid-rigid solid interface	45
1.5.3 Dynamical boundary conditions at a fluid-fluid interface	46
1.5.4 Boundary conditions on temperature	57
1.6 Dimensionless Parameters	57
1.7 Special Forms of the Governing Equations	60
1.7.1 First integral of the momentum equation	60

1.7.2	Bernoulli's equation	71
1.7.3	Kelvin's circulation theorem	75
1.7.4	Helmholtz's vorticity equation	77
1.8	Example Applications	78
1.8.1	Flow past a cylinder	79
1.8.2	von Karman momentum integral	81
1.8.3	Flow of inviscid fluid over a bump	84
1.8.4	Jet impacting on a moving plate	89
1.8.5	Rocket propulsion	93
1.8.6	Impulse turbine	94
1.8.7	Slip-stream analysis of a wind turbine	99
1.8.8	Flow through a pipe network	101
2	Hydrostatics	109
2.1	Force on a Plane Surface	110
2.2	Forces on a Curved Surface	113
2.3	Example Applications	116
3	Ideal Fluid Flow	126
3.1	Simplification of the Euler Equations for Potential Flow	127
3.2	Two-dimensional Potential Flow	127
3.3	Circulation and Volume Flow Rate	132
3.4	Elementary Complex Potentials	133
3.5	Rankine Oval	140
3.6	Uniform Flow Past a Cylinder	143
3.7	Method of Images	145
3.8	Blasius Force Theorems	149
3.9	Flow Past a Cylinder with Point Vortex at the Origin	154
3.10	Circle Theorem	158
3.11	Conformal Mapping	161
3.12	Free-Streamline Theory	169
3.13	Two-Dimensional Flow with Vorticity	174
3.14	Three-dimensional Potential Flow	176
3.15	Elementary Potentials	181
3.16	Force on a Rigid Body in Potential Flow	184
3.17	Flow Around a Sphere	185
3.18	Kinetic Energy of a Moving Fluid	187
3.19	Motion of a Sphere Through a Stationary Fluid	190
3.20	Motion of a Bubble through a Stationary Fluid	192

4	Surface Waves	204
4.1	Governing Equations for Surface Waves	204
4.2	Small Amplitude Two-Dimensional Waves	206
4.3	Small-Amplitude Traveling Waves	208
4.3.1	Particle paths for traveling waves	210
4.3.2	Stream function for traveling waves	212
4.4	The Dispersion Relation	212
4.5	Standing Waves	214
4.6	Traveling Waves at the Interface of Two Liquids	214
4.7	Group Velocity	218
4.8	Shallow-Water Finite Waves	221
4.8.1	The wave equation	223
4.8.2	Finite-amplitude waves: The method of characteristics	227
4.9	The Dam Breaking Problem	232
4.9.1	The dam break problem without tailwater	232
4.9.2	Shock waves	235
4.9.3	Dam break problem with tailwater	239
5	Exact Solutions to Flow Problems of an Incompressible Viscous Fluid	244
5.1	Governing Equations	245
5.2	Plane Steady Unidirectional Flows	247
5.3	Poiseuille Flow	252
5.4	Stagnation-Point Flow	263
5.5	Flow in a Converging or Diverging Channel	266
5.6	Flow between Rotating Cylinders	269
5.7	Fluid in a Rotating Body	272
5.8	Flow over an Infinite Rotating Disc	273
5.9	Unsteady Flow Problems	274
5.9.1	Stokes' first problem	274
5.9.2	Stokes' second problem	279
5.9.3	Radial growth or collapse of a spherical bubble	281
5.9.4	Unidirectional flow through a circular pipe	282
5.9.5	Transient flow inside or outside a rotating cylinder	291
5.9.6	Squeeze-film flows	294
5.10	Wind Driven Ocean Currents: The Ekman Layer	302
6	Laminar Boundary Layer Theory	318
6.1	Governing Equations	319
6.2	Blasius Solution	322
6.3	Falkner-Skan Solution	326

6.4	Displacement and Momentum Thickness	330
6.5	Approximate Techniques: The von Karman Momentum Integral	332
6.6	Thermal Boundary Layer	336
7	Low-Reynolds Number Hydrodynamics	345
7.1	Lubrication Theory	345
7.2	Low-Inertia External Flows	349
7.3	Three-Dimensional Axisymmetric Flows	350
7.3.1	Solution using separation of variables	352
7.3.2	Uniform flow around a sphere	356
7.3.3	Motion of a spherical bubble in a fluid	357
7.4	Flow Between Rotating Spheres	362
7.5	Uniform Flow Around a Circular Cylinder	364
8	Compressible Fluid Flow	367
8.1	One-Dimensional Flow Equations	368
8.2	Acoustic Theory: The Wave Equation	371
8.3	Isentropic Flow Relations for a One-Dimensional Steady Flow Field	373
8.4	Solution of the One-Dimensional Flow Equations	375
8.5	Normal Shock	381
8.5.1	Stationary normal shock	381
8.5.2	Moving shock wave	384
8.5.3	Structure of a normal shock	387
8.6	Oblique Shock and Expansion Waves	391
8.6.1	Mach waves	392
8.6.2	Oblique shock relations	393
8.6.3	Prandtl-Meyer expansion waves	403
8.7	Shock-Expansion Theory	407
8.8	Isentropic Flow through Variable-Area Ducts	410
8.9	Area-Velocity Relation	412
8.10	Flow Through a Convergent-Divergent Nozzle	415
8.11	Unsteady One-Dimensional Flow Problems: Finite Waves	423
8.12	The Shock Tube Problem	425
8.13	Compressible Couette Flow	428
A	Equations of Motion in Cylindrical Coordinates	433
B	Equations of Motion in Spherical Coordinates	435

C The Laplace Transform Method	437
Bibliography	440
Answers and Hints to Selected Exercises	443
Index	458