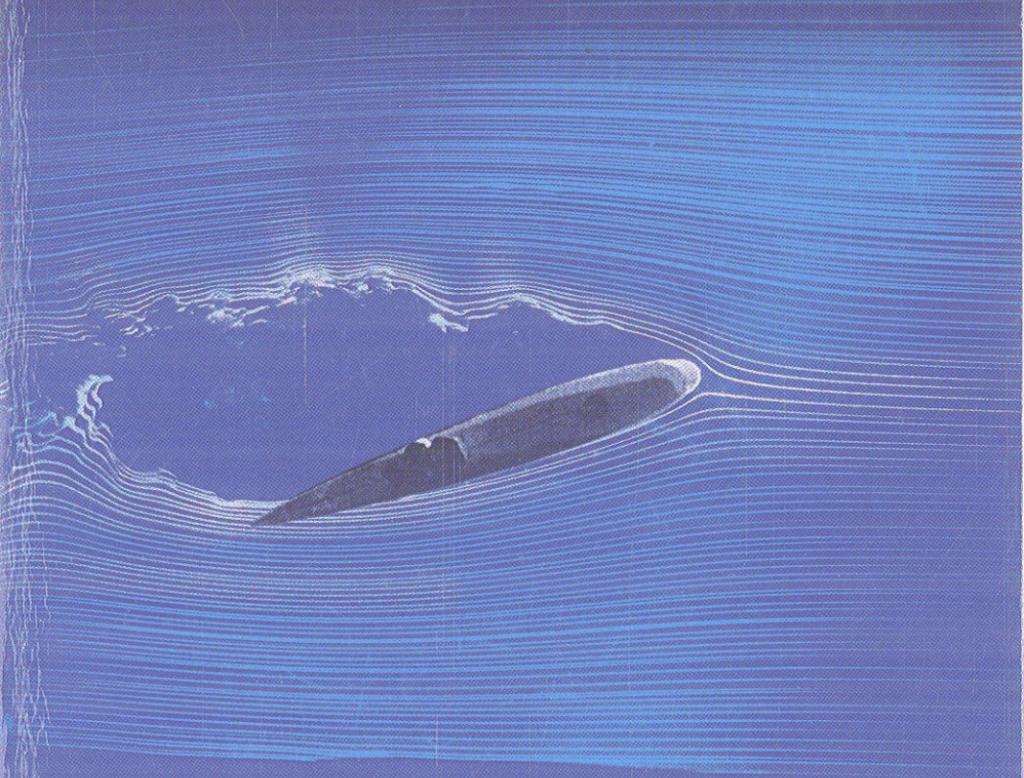


# *Basic Wing and Airfoil Theory*



Alan Pope

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