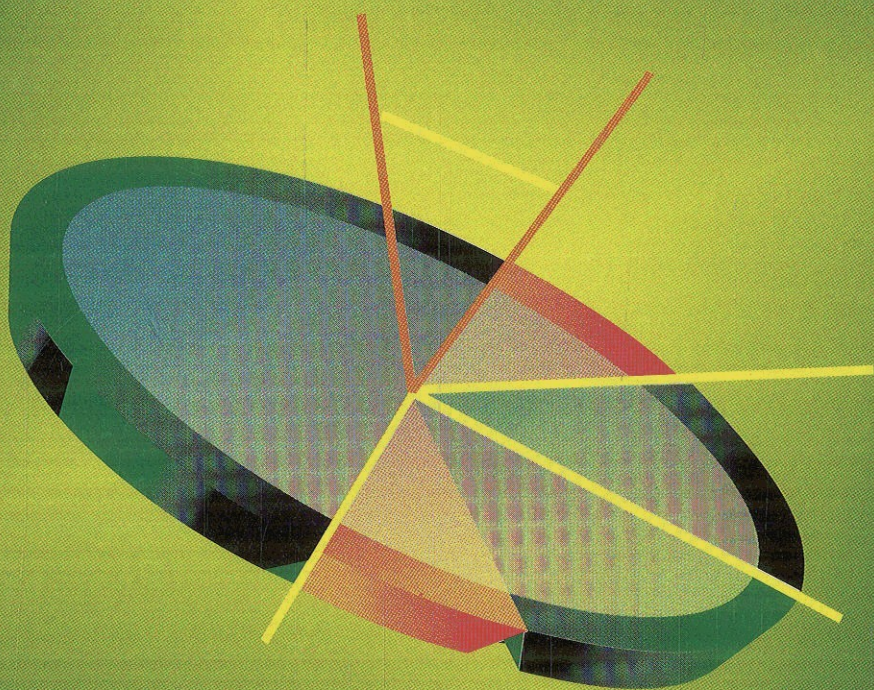


Vector Analysis with Applications



Alpha
Science

Absos Ali Shaikh
Sanjib Kumar Jana

Contents

Preface

vii

I. Elements of Vector Algebra	1
I.0 Introduction	1
I.1 Scalars and Vectors	2
I.1.1 Localised and Free Vector	5
I.2 Equality of Vectors	6
I.3 Multiplication of Vector by a Scalar	6
I.4 Addition and Subtraction of Vectors	7
I.4.1 Addition of Several Vectors	9
I.4.2 Commutative and Associative Laws of Addition	10
I.5 Collinear and Coplanar Vectors	11
I.6 Linearly Dependent and Independent Set of Vectors	11
I.7 Vector Space	14
I.8 Position Vector	15
I.9 Point of Division	15
I.10 Composition and Resolution of Vectors	17
I.11 Orthogonal Resolution of a Vector	18
I.12 Worked Out Examples	19
I.13 Exercises	41
 II. Vector Products	 45
II.0 Introduction	45
II.1 Scalar Product	45
II.1.1 Geometrical Interpretation of Scalar Product	46
II.2 Properties of Scalar Product	47
II.3 Linear Independence and Orthogonality of Vectors	49
II.4 Vector Product	52
II.4.1 Geometrical Interpretation of Vector Product	53
II.5 Properties of Vector Product	55
II.6 Scalar Triple Product	58
II.6.1 Geometrical Interpretation of Scalar Triple Product	59
II.6.2 Conditions for Vanishing of $[\vec{x}, \vec{y}, \vec{z}]$	60
II.7 Vector Triple Product	61
II.8 Volume of a Tetrahedron	63
II.9 Product of Four Vectors	63
II.9.1 Scalar Product of Four Vectors	64

II.9.2 Vector Product of Four Vectors	64
II.10 Reciprocal Systems of Vectors	65
II.11 Worked Out Examples	66
II.12 Exercises	94
III.Applications of Vector Algebra	98
III.0 Introduction	98
III.1 Vector Equation	98
III.2 Straight Line	102
III.2.1 Straight Line Through a Point and Parallel to a Vector . . .	102
III.2.2 Straight Line Through Two Points	103
III.2.3 Straight Line Through a Point and Perpendicular to Two Vec- tors	105
III.2.4 Intersection of Two Straight Lines	105
III.2.5 Angular Bisector of Two Intersecting Straight Lines	106
III.2.6 <i>Distance of a Point From a Straight Line</i>	107
III.3 Plane	108
III.3.1 Plane Through a Point and Parallel to Two Vectors	108
III.3.2 Plane Through Two Points and Parallel to a Vector	109
III.3.3 Plane Through Three Points	110
III.3.4 Plane Through a Point and Perpendicular to a Vector	111
III.3.5 Angle Between Two Planes	112
III.3.6 Distance of a Point From a Plane	113
III.3.7 Plane Through the Intersection of Two Planes	113
III.3.8 Plane Containing a Line and Parallel to a Vector	114
III.3.9 Plane Containing a Line and Passing Through a Vector . . .	115
III.4 Shortest Distance Between Two Skew Lines	116
III.5 Sphere	117
III.5.1 Sphere with Given Radius and Centre	117
III.5.2 Sphere with Given Extremities of a Diameter	118
III.5.3 Intersection of a Line and a Sphere	118
III.5.4 Tangent Plane to a Sphere	119
III.6 Centroid, Mass Vector and Centre of Mass	120
III.7 Force	123
III.7.1 Resultant of Concurrent Forces	123
III.7.2 Resolution of Forces	125
III.7.3 Equilibrium of Coplanar Concurrent Forces	126
III.7.4 Relative Velocity	129
III.7.5 Work, Power	130
III.7.6 Moment of a Force (Torque)	131
III.7.7 Linear Velocity	132
III.8 Worked Out Examples	133
III.9 Exercises	155

IV. Vector Differential Calculus	161
IV.0 Introduction	161
IV.1 Vector Function	161
IV.1.1 Geometrical Interpretation of Vector Function	163
IV.1.2 Scalar and Vector Fields and Point Functions	163
IV.2 Limit, Continuity	165
IV.3 Derivative of a Vector Function	170
IV.3.1 Geometrical Interpretation of Derivative of a Vector Function	171
IV.3.2 Successive Differentiation	184
IV.3.3 Partial Differentiation	185
IV.4 Vector Differential Operators	188
IV.5 Gradient	189
IV.5.1 Geometrical Interpretation of Gradient	189
IV.6 Directional Derivative	194
IV.7 Divergence	195
IV.7.1 Physical Interpretation of Divergence	196
IV.8 Curl	200
IV.8.1 Physical Interpretation of Curl	201
IV.9 Laplacian	205
IV.10 Worked Out Examples	208
IV.11 Exercises	240
V. Applications of Vector Differential Calculus	247
V.0 Introduction	247
V.1 Space Curve	247
V.2 Tangent and Normal	248
V.2.1 Geometrical Interpretation of Curvature of a Plane Curve	250
V.3 Normal Plane, Osculating Plane and Rectifying Plane	252
V.4 Frenet-Serret Formulae	255
V.5 Intrinsic Equation	258
V.6 Surface	259
V.7 Velocity and Acceleration	259
V.7.1 Tangential and Normal Acceleration	260
V.7.2 Radial and Cross Radial Acceleration	261
V.7.3 Velocity and Acceleration Parallel to Rotating Axes	262
V.7.4 Uniform Motion on a Circle	264
V.8 Relative Motion	265
V.9 Areal Velocity	265
V.10 Worked Out Examples	267
V.11 Exercises	292
VI. Vector Integral Calculus and Differential Equations	297
VI.0 Introduction	297
VI.1 Vector Integration	297
VI.1.1 Indefinite Integration	297
VI.1.2 General Rules of Indefinite Integration	299

VI.1.3 Definite Integration	301
VI.2 Some Useful Definitions	303
VI.3 Line Integral	305
VI.3.1 Physical Interpretation of the Line Integral	307
VI.4 Surface Integral	313
VI.4.1 Cartesian Form of the Surface Integral	314
VI.4.2 Physical Interpretation of the Surface Integral	315
VI.5 Volume Integral	315
VI.6 Green's Theorem	316
VI.6.1 Green's Theorem for a Multiply Connected Region	319
VI.7 Stokes' Theorem	321
VI.8 Gauss' Divergence Theorem	324
VI.9 Alternative Definitions of Gradient, Divergence and Curl	326
VI.10 Vector Differential Equations	329
VI.10.1 Formation of Vector Differential Equation	331
VI.10.2 Solution of Vector Differential Equation	331
VI.11 Worked Out Examples.	333
VI.12 Exercises	384

VII. Applications of Vector Calculus 391

VII.0 Introduction	391
VII.1 Dynamics of a Particle	391
VII.1.1 Impulse	392
VII.1.2 Work	393
VII.1.3 Central Force	394
VII.1.4 Motion Under Inverse Square Law	395
VII.1.5 Deduction of Differential Equation of the Central Orbit	397
VII.2 Dynamics of Rigid Bodies	398
VII.2.1 D'Alembert's Principle	399
VII.2.2 General Equations of Motion of a Rigid Body	399
VII.2.3 Center of Inertia	401
VII.2.4 Linear and Angular Momentum of a System of Particles	403
VII.2.5 Kinetic Energy of a System of Particles	404
VII.2.6 Lagrange's Equations	405
VII.3 Electrostatics	407
VII.3.1 Electric Field	408
VII.3.2 Gauss's Law	410
VII.3.3 Deduction of Coulomb's Law From Gauss's Law	411
VII.4 Magnetism	412
VII.4.1 Biot-Savart Law	413
VII.4.2 Divergence of $\vec{B}$	415
VII.4.3 Curl of $\vec{B}$	415
VII.4.4 Magnetic Vector and Scalar Potential	416
VII.5 Fluid Mechanics	417
VII.6 Microbiology	418
VII.7 Worked Out Examples	419

VII.8 Exercises	430
---------------------------	-----

VIII. Historical Background	433
------------------------------------	------------

Bibliography	438
---------------------	------------

Index	439
--------------	------------