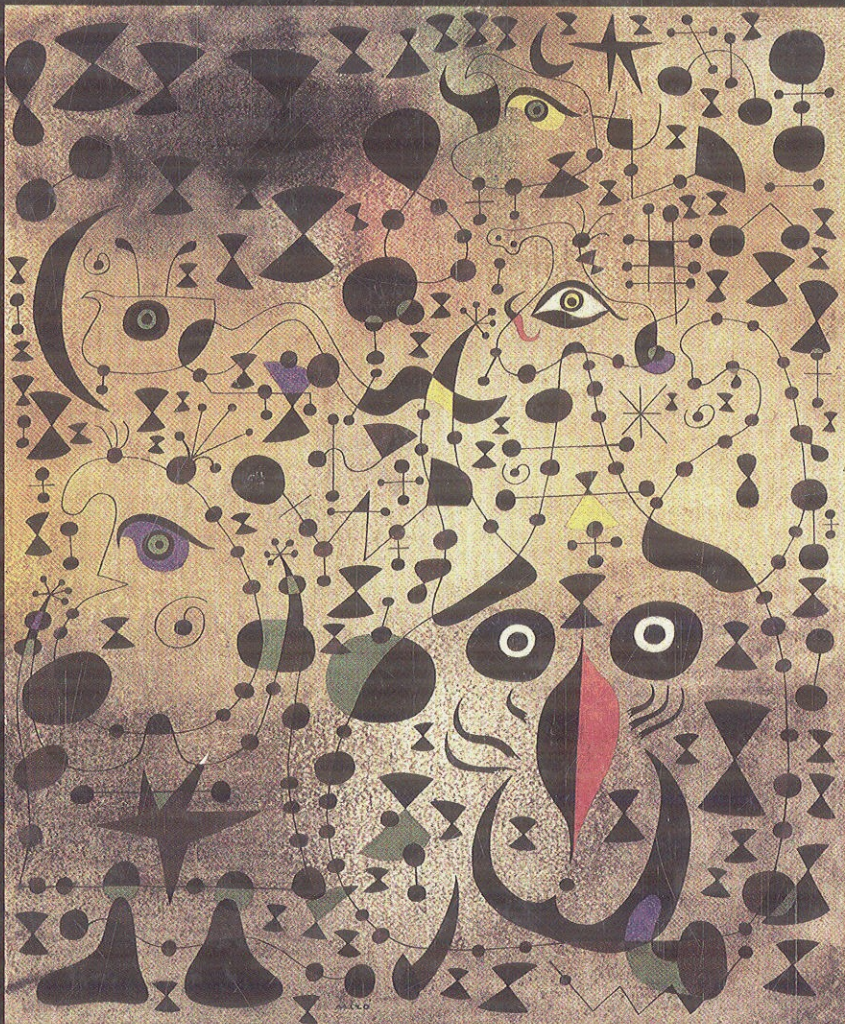


THOMAS BANKS

# Modern Quantum Field Theory

A Concise Introduction



CAMBRIDGE

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