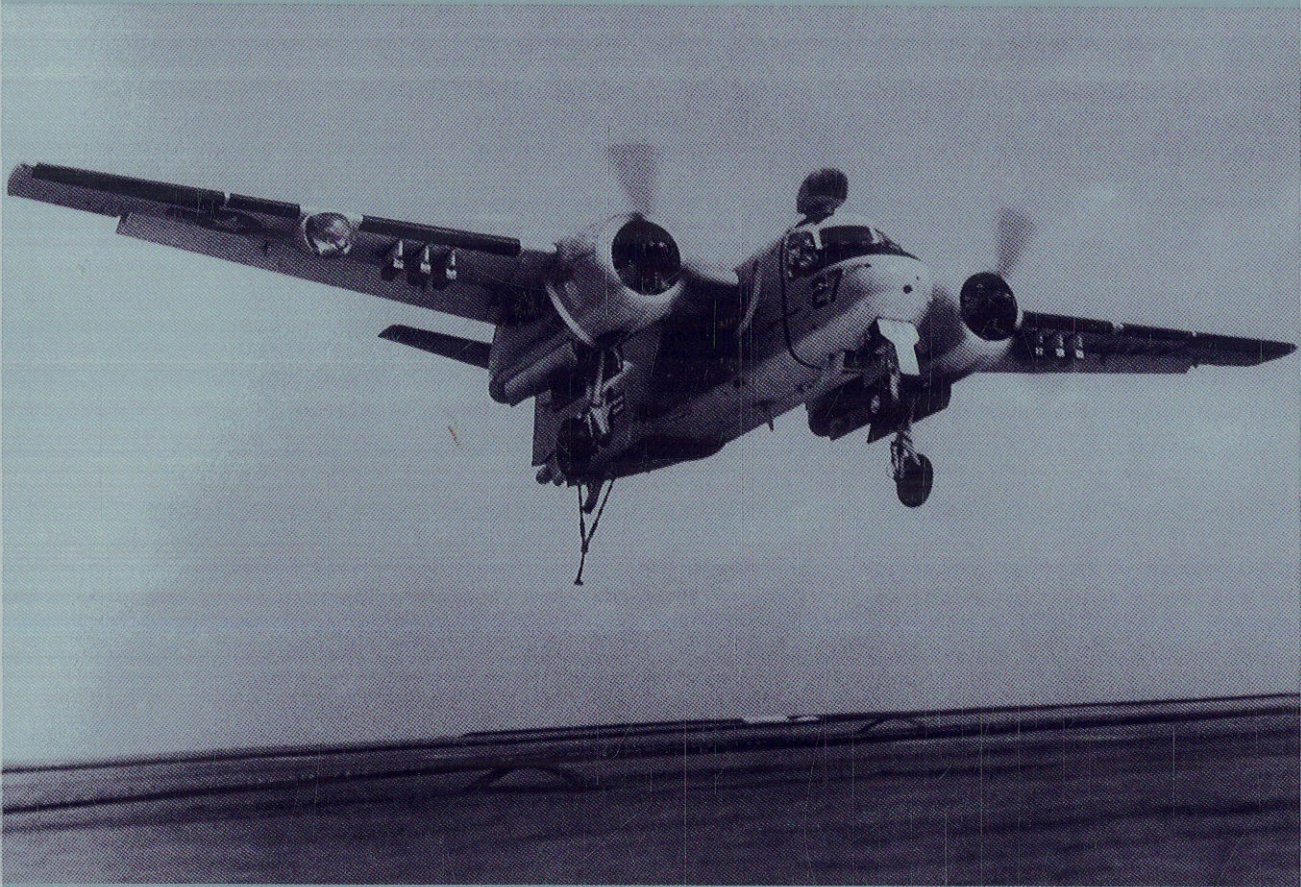


# Introduction to Structural Dynamics



Bruce K. Donaldson

CAMBRIDGE

# Contents

Preface for the Student	page xi
Preface for the Instructor	xv
Acknowledgments	xvii
List of Symbols	xix
<b>1 The Lagrange Equations of Motion . . . . .</b>	<b>1</b>
1.1 Introduction	1
1.2 Newton's Laws of Motion	2
1.3 Newton's Equations for Rotations	5
1.4 Simplifications for Rotations	8
1.5 Conservation Laws	12
1.6 Generalized Coordinates	12
1.7 Virtual Quantities and the Variational Operator	15
1.8 The Lagrange Equations	19
1.9 Kinetic Energy	25
1.10 Summary	29
Chapter 1 Exercises	33
Endnote (1): Further Explanation of the Variational Operator	37
Endnote (2): Kinetic Energy and Energy Dissipation	41
Endnote (3): A Rigid Body Dynamics Example Problem	42
<b>2 Mechanical Vibrations: Practice Using the Lagrange Equations . . . . .</b>	<b>46</b>
2.1 Introduction	46
2.2 Techniques of Analysis for Pendulum Systems	47
2.3 Example Problems	53
2.4 Interpreting Solutions to Pendulum Equations	66
2.5 Linearizing Differential Equations for Small Deflections	71
2.6 Summary	72
2.7 <b>**Conservation of Energy versus the Lagrange Equations**</b>	<b>73</b>
2.8 <b>**Nasty Equations of Motion**</b>	<b>80</b>
2.9 <b>**Stability of Vibratory Systems**</b>	<b>82</b>
Chapter 2 Exercises	85

Endnote (1): The Large-Deflection, Simple Pendulum Solution	93
Endnote (2): Divergence and Flutter in Multidegree of Freedom, Force Free Systems	94
<b>3 Review of the Basics of the Finite Element Method for Simple Elements . . . . .</b>	<b>99</b>
3.1 Introduction	99
3.2 Generalized Coordinates for Deformable Bodies	100
3.3 Element and Global Stiffness Matrices	103
3.4 More Beam Element Stiffness Matrices	112
3.5 Summary	123
Chapter 3 Exercises	133
Endnote (1): A Simple Two-Dimensional Finite Element	138
Endnote (2): The Curved Beam Finite Element	146
<b>4 FEM Equations of Motion for Elastic Systems . . . . .</b>	<b>157</b>
4.1 Introduction	157
4.2 Structural Dynamic Modeling	158
4.3 Isolating Dynamic from Static Loads	163
4.4 Finite Element Equations of Motion for Structures	165
4.5 Finite Element Example Problems	172
4.6 Summary	186
4.7 <b>**Offset Elastic Elements**</b>	193
Chapter 4 Exercises	195
Endnote (1): Mass Refinement Natural Frequency Results	205
Endnote (2): The Rayleigh Quotient	206
Endnote (3): The Matrix Form of the Lagrange Equations	210
Endnote (4): The Consistent Mass Matrix	210
Endnote (5): A Beam Cross Section with Equal Bending and Twisting Stiffness Coefficients	211
<b>5 Damped Structural Systems . . . . .</b>	<b>213</b>
5.1 Introduction	213
5.2 Descriptions of Damping Forces	213
5.3 The Response of a Viscously Damped Oscillator to a Harmonic Loading	230
5.4 Equivalent Viscous Damping	239
5.5 Measuring Damping	242
5.6 Example Problems	243
5.7 Harmonic Excitation of Multidegree of Freedom Systems	247
5.8 Summary	248
Chapter 5 Exercises	253
Endnote (1): A Real Function Solution to a Harmonic Input	260
<b>6 Natural Frequencies and Mode Shapes . . . . .</b>	<b>263</b>
6.1 Introduction	263

6.2	Natural Frequencies by the Determinant Method	265
6.3	Mode Shapes by Use of the Determinant Method	273
6.4	<b>**Repeated Natural Frequencies**</b>	279
6.5	Orthogonality and the Expansion Theorem	289
6.6	The Matrix Iteration Method	293
6.7	<b>**Higher Modes by Matrix Iteration**</b>	300
6.8	Other Eigenvalue Problem Procedures	307
6.9	Summary	311
6.10	<b>**Modal Tuning**</b>	315
	Chapter 6 Exercises	320
	Endnote (1): Linearly Independent Quantities	323
	Endnote (2): The Cholesky Decomposition	324
	Endnote (3): Constant Momentum Transformations	326
	Endnote (4): Illustration of Jacobi's Method	329
	Endnote (5): The Gram–Schmidt Process for Creating Orthogonal Vectors	332
<b>7</b>	<b>The Modal Transformation . . . . .</b>	<b>334</b>
7.1	Introduction	334
7.2	Initial Conditions	334
7.3	The Modal Transformation	337
7.4	Harmonic Loading Revisited	340
7.5	Impulsive and Sudden Loadings	342
7.6	The Modal Solution for a General Type of Loading	351
7.7	Example Problems	353
7.8	Random Vibration Analyses	363
7.9	Selecting Mode Shapes and Solution Convergence	366
7.10	Summary	371
7.11	<b>**Aeroelasticity**</b>	373
7.12	<b>**Response Spectrums**</b>	388
	Chapter 7 Exercises	391
	Endnote (1): Verification of the Duhamel Integral Solution	396
	Endnote (2): A Rayleigh Analysis Example	398
	Endnote (3): An Example of the Accuracy of Basic Strip Theory	399
	Endnote (4): Nonlinear Vibrations	400
<b>8</b>	<b>Continuous Dynamic Models . . . . .</b>	<b>402</b>
8.1	Introduction	402
8.2	Derivation of the Beam Bending Equation	402
8.3	Modal Frequencies and Mode Shapes for Continuous Models	406
8.4	Conclusion	431
	Chapter 8 Exercises	438
	Endnote (1): The Long Beam and Thin Plate Differential Equations	439
	Endnote (2): Derivation of the Beam Equation of Motion Using Hamilton's Principle	442

Endnote (3): Sturm–Liouville Problems	445
Endnote (4): The Bessel Equation and Its Solutions	445
Endnote (5): Nonhomogeneous Boundary Conditions	449
<b>9 Numerical Integration of the Equations of Motion . . . . .</b>	<b>451</b>
9.1 Introduction	451
9.2 The Finite Difference Method	452
9.3 Assumed Acceleration Techniques	460
9.4 Predictor-Corrector Methods	463
9.5 The Runge-Kutta Method	468
9.6 Summary	474
9.7 <b>**Matrix Function Solutions**</b>	<b>475</b>
Chapter 9 Exercises	480
<b>Appendix I. Answers to Exercises . . . . .</b>	<b>483</b>
Chapter 1 Solutions	483
Chapter 2 Solutions	486
Chapter 3 Solutions	494
Chapter 4 Solutions	498
Chapter 5 Solutions	509
Chapter 6 Solutions	516
Chapter 7 Solutions	519
Chapter 8 Solutions	525
Chapter 9 Solutions	529
<b>Appendix II. Fourier Transform Pairs . . . . .</b>	<b>531</b>
II.1 Introduction to Fourier Transforms	531
Index	537