

Latif M. Jiji



Heat Conduction

Third Edition

 Springer

CONTENTS

Preface

vii

CHAPTER 1: BASIC CONCEPTS	1
1.1 Examples of Conduction Problems	1
1.2 Focal Point in Conduction Heat Transfer	2
1.3 Fourier's Law of Conduction	2
1.4 Conservation of Energy: Differential Formulation of the Heat Conduction in Rectangular Coordinates	5
1.5 The Heat Conduction Equation in Cylindrical and Spherical Coordinates	9
1.6 Boundary Conditions	10
1.6.1 Surface Convection: Newton's Law of Cooling	10
1.6.2 Surface Radiation: Stefan-Boltzmann Law	11
1.6.3 Examples of Boundary Conditions	12
1.7 Problem Solving Format	15
1.8 Units	16
REFERENCES	17
PROBLEMS	18
CHAPTER 2: ONE-DIMENSIONAL STEADY-STATE CONDUCTION	24
2.1 Examples of One-dimensional Conduction	24
2.2 Extended Surfaces: Fins	34
2.2.1 The Function of Fins	34
2.2.2 Types of Fins	34
2.2.2 Heat Transfer and Temperature Distribution in Fins	35
2.2.4 The Fin Approximation	36
2.2.5 The Fin Heat Equation: Convection at Surface	37
2.2.6 Determination of dA_s / dx	39
2.2.7 Boundary Conditions	40
2.2.8 Determination of Fin Heat Transfer Rate q_f	40
2.2.9 Steady State Applications: Constant Area Fins with Surface Convection	41

2.2.10	Corrected Length L_c	44
2.2.11	Fin Efficiency η_f	44
2.2.12	Moving Fins	45
2.2.13	Application of Moving Fins	47
2.2.14	Variable Area Fins	49
2.3	Bessel Differential Equations and Bessel Functions	52
2.3.1	General Form of Bessel Equations	52
2.3.2	Solutions: Bessel Functions	52
2.3.3	Forms of Bessel Functions	54
2.3.4	Special Closed-form Bessel Functions: $n = \text{odd integer}/2$	54
2.3.5	Special Relations for $n = 1, 2, 3, \dots$	55
2.3.6	Derivatives and Integrals of Bessel Functions	56
2.3.7	Tabulation and Graphical Representation of Selected Bessel Functions	56
2.4	Equidimensional (Euler) Equation	58
2.5	Graphically Presented Solutions to Fin Heat Transfer Rate q_f	59
	REFERENCES	60
	PROBLEMS	61

CHAPTER 3: TWO-DIMENSIONAL STEADY STATE CONDUCTION		72
3.1	The Heat Conduction Equation	72
3.2	Method of Solution and Limitations	72
3.3	Homogeneous Differential Equations and Boundary Conditions	72
3.4	Sturm-Liouville Boundary-Value Problem: Orthogonality	74
3.5	Procedure for the Application of Separation of Variables Method	76
3.6	Cartesian Coordinates: Examples	83
3.7	Cylindrical Coordinates: Examples	97
3.8	Integrals of Bessel Functions	102
3.9	Non-homogeneous Differential Equations	103
3.10	Non-homogeneous Boundary Conditions: The Method of Superposition	109
	REFERENCES	111

PROBLEMS	111
CHAPTER 4: TRANSIENT CONDUCTION	119
4.1 Simplified Model: Lumped-Capacity Method	119
4.1.1 Criterion for Neglecting Spatial Temperature Variation	119
4.1.2 Lumped-Capacity Analysis	121
4.2 Transient Conduction in Plates	124
4.3 Non-homogeneous Equations and Boundary Conditions	128
4.4 Transient Conduction in Cylinders	132
4.5 Transient Conduction in Spheres	138
4.6 Time Dependent Boundary Conditions: Duhamel's Superposition Integral	141
4.6.1 Formulation of Duhamel's Integral	142
4.6.2 Extension to Discontinuous Boundary Conditions	144
4.6.3 Applications	145
4.7 Conduction in Semi-infinite Regions: The Similarity Transformation Method	150
REFERENCES	154
PROBLEMS	154
CHAPTER 5: POROUS MEDIA	163
5.1 Examples of Conduction in Porous Media	163
5.2 Simplified Heat Transfer Model	164
5.2.1 Porosity	164
5.2.2 Heat Conduction Equation: Cartesian Coordinates	165
5.2.3 Boundary Conditions	167
5.2.4 Heat Conduction Equation: Cylindrical Coordinates	168
5.3 Applications	168
REFERENCES	174
PROBLEMS	175

CHAPTER 6: CONDUCTION WITH PHASE CHANGE: MOVING BOUNDARY PROBLEMS	184
6.1 Introduction	184
6.2 The Heat Equation	185
6.3 Moving Interface Boundary Conditions	185
6.4 Non-linearity of the Interface Energy Equation	188
6.5 Non-dimensional Form of the Governing Equations: Governing Parameters	189
6.6 Simplified Model: Quasi-Steady Approximation	190
6.7 Exact Solutions	197
6.7.1 Stefan's Solution	197
6.7.2 Neumann's Solution: Solidification of Semi- Infinite Region	200
6.7.3 Neumann's Solution: Melting of Semi-Infinite Region	203
6.8 Effect of Density Change on the Liquid Phase	204
6.9 Radial Conduction with Phase Change	205
6.10 Phase Change in Finite Regions	209
REFERENCES	210
PROBLEMS	210

CHAPTER 7: NON-LINEAR CONDUCTION PROBLEMS	215
7.1 Introduction	215
7.2 Sources of Non-linearity	215
7.2.1 Non-linear Differential Equations	215
7.2.2 Non-linear Boundary Conditions	216
7.3 Taylor Series Method	216
7.4 Kirchhoff Transformation	220
7.4.1 Transformation of Differential Equations	220
7.4.2 Transformation of Boundary Conditions	221
7.5 Boltzmann Transformation	224
7.6 Combining Boltzmann and Kirchhoff Transformations	226
7.7 Exact Solutions	227
REFERENCES	230
PROBLEMS	230

CHAPTER 8: APPROXIMATE SOLUTIONS: THE INTEGRAL METHOD	236
8.1 Integral Method Approximation: Mathematical Simplification	236
8.2 Procedure	236
8.3 Accuracy of the Integral Method	237
8.4 Application to Cartesian Coordinates	238
8.5 Application to Cylindrical Coordinates	246
8.6 Non-linear Problems	251
8.7 Energy Generation	260
REFERENCES	264
PROBLEMS	264
 CHAPTER 9: PERTURBATION SOLUTIONS	 269
9.1 Introduction	269
9.2 Solution Procedure	270
9.3 Examples of Perturbation Problems in Conduction	271
9.4 Perturbation Solutions: Examples	273
9.5 Useful Expansions	296
REFERENCES	296
PROBLEMS	297
 CHAPTER 10: HEAT TRANSFER IN LIVING TISSUE	 302
10.1 Introduction	302
10.2 Vascular Architecture and Blood Flow	302
10.3 Blood Temperature Variation	304
10.4 Mathematical Modeling of Vessels-Tissue Heat Transfer	305
10.4.1 Pennes Bioheat Equation	305
10.4.2 Chen-Holmes Equation	312
10.4.3 Three-Temperature Model for Peripheral Tissue	313
10.4.4 Weinbaum-Jiji Simplified Bioheat Equation for Peripheral Tissue	315
10.4.5 The s -Vessel Tissue Cylinder Model	323
REFERENCES	332
PROBLEMS	334

CHAPTER 11: MICROSCALE CONDUCTION	347
11.1 Introduction	347
11.1.1 Categories of Microscale Phenomena	348
11.1.2 <i>Purpose and Scope of this Chapter</i>	350
11.2 Understanding the Essential Physics of Thermal Conductivity Using the Kinetic Theory of Gases	351
11.2.1 Determination of Fourier's Law and Expression for Thermal Conductivity	351
11.3 Energy Carriers	355
11.3.1 Ideal Gas: Heat is Conducted by Gas Molecules	355
11.3.2 Metals: Heat is Conducted by Electrons	359
11.3.3 Electrical Insulators and Semiconductors: Heat is Conducted by Phonons (Sound Waves)	361
11.3.4 Radiation: Heat is Carried by Photons (Light Waves)	372
11.4 Thermal Conductivity Reduction by Boundary Scattering: The Classical Size Effect	376
11.4.1 Accounting for Multiple Scattering Mechanisms: Matthiessen's Rule	377
11.4.2 Boundary Scattering for Heat Flow Parallel to Boundaries	379
11.4.3 Boundary Scattering for Heat Flow Perpendicular to Boundaries	387
11.5 Closing Thoughts	391
REFERENCES	394
PROBLEMS	397
APPENDIX A: Ordinary Differential Equations	402
(1) Second Order Differential Equations with Constant Coefficients	402
(2) First Order Ordinary Differential Equations with Variable Coefficients	404
APPENDIX B: Integrals of Bessel Functions	405
APPENDIX C: Values of Bessel Functions	406
APPENDIX D: Fundamental Physical Constants and Material Properties	412
D-1 Fundamental Physical Constants	412
D-2 Unit conversions	412

D-3 Properties of Helium Gas	412
D-4 Properties of Copper at 300 K	412
D-5 Properties of Fused Silica (Amorphous Silicon Dioxide, SiO ₂) at 300 K	412 413
D-6 Properties of Silicon	413
D-7 Measured Thermal Conductivity of a 56 nm Diameter Silicon Nanowire at Selected Temperatures	414
D-8 Calculated Thermal Conductivity of Single-Walled Carbon Nanotubes, Selected Values	414
INDEX	416