NONLINEAR
PHYSICAL
SCIENCE

Albert C. J. Luo

Discontinuous Dynamical Systems on Time-varying Domains

高等教育出版社
HIGHER EDUCATION PRESS

Springer

Contents

1 **Introduction** 1
1.1 Discontinuous systems 1
1.2 Book layout 5
References 6

2 **Flow Switchability** 9
2.1 Discontinuous dynamic systems 9
2.2 *G*-functions 11
2.3 Passable flows 15
2.4 Non-passable flows 19
2.5 Tangential flows 28
2.6 Switching bifurcations 39
References 52

3 **Transversality and Sliding Phenomena** 55
3.1 A controlled system 55
3.2 Transversality conditions 57
3.3 Mappings and predictions 60
3.4 Periodic and chaotic motions 67
References 75

4 **A Frictional Oscillator on Time-varying Belt** 77
4.1 Mechanical model 77
4.2 *Analytical conditions* 80
4.2.1 Equations of motion 80
4.2.2 Passable flows to boundary 83
4.2.3 Sliding flows on boundary 85
4.2.4 Grazing flows to boundary 89
4.3 Generic mappings and force product criteria 91
4.3.1 Generic mappings 91

4.3.2 Sliding flows and fragmentation ... 93
4.3.3 Grazing flows ... 96
4.4 Periodic motions ... 98
4.4.1 Mapping structures ... 98
4.4.2 Illustrations ... 100
4.5 Numerical simulations ... 112
References ... 113

5 Two Oscillators with Impacts and Stick ... 115
5.1 Physical problem ... 115
5.1.1 Introduction to problem ... 115
5.1.2 Equations of motion ... 117
5.2 Domains and vector fields ... 119
5.2.1 Absolute motion description ... 119
5.2.2 Relative motion description ... 125
5.3 Mechanism of stick and grazing ... 127
5.3.1 Analytical conditions ... 128
5.3.2 Physical interpretation ... 133
5.4 Mapping structures and motions ... 134
5.4.1 Switching sets and basic mappings ... 134
5.4.2 Mapping equations ... 137
5.4.3 Mapping structures ... 141
5.4.4 Bifurcation scenario ... 144
5.5 Periodic motion prediction ... 144
5.5.1 Approach ... 145
5.5.2 Impacting chatter ... 147
5.5.3 Impacting chatter with stick ... 150
5.5.4 Parameter maps ... 152
5.6 Numerical illustrations ... 153
5.6.1 Impacting chatter ... 153
5.6.2 Impacting chatter with stick ... 155
5.6.3 Further illustrations ... 158
References ... 160

6 Dynamical Systems with Frictions ... 161
6.1 Problem statement ... 161
6.2 Switching and stick motions ... 164
6.2.1 Equations of motion ... 164
6.2.2 Analytical conditions ... 166
6.3 Periodic motions ... 172
6.3.1 Switching planes and mappings ... 172
6.3.2 Mapping structures and motions ... 175
6.3.3 Bifurcation scenario ... 178
6.4 Numerical illustrations ... 181
6.4.1 Periodic motion without stick ... 181

6.4.2 Periodic motion with stick 184
6.4.3 Periodic motion with stick only 189
References 189

7 Principles for System Interactions 191
7.1 Two dynamical systems 191
7.1.1 Dynamical systems with interactions 191
7.1.2 Discontinuous description 195
7.1.3 Resultant dynamical systems 196
7.2 Fundamental interactions 199
7.3 Interactions with singularity 206
7.4 Interactions with corner singularity 210
References 215

Appendix 217
A.1 Basic solution 217
A.2 Stability and bifurcation 219

Index 221