

Advances in Fuzzy Systems — Applications and Theory – Vol. 24

Nonlinear Integrals and Their Applications in Data Mining

Zhenyuan Wang

Rong Yang

Kwong-Sak Leung

Contents

Preface	vii
List of Tables	xv
List of Figures	xvi
Chapter 1: Introduction	1
Chapter 2: Basic Knowledge on Classical Sets	4
2.1 Classical Sets and Set Inclusion	4
2.2 Set Operations	7
2.3 Set Sequences and Set Classes	10
2.4 Set Classes Closed Under Set Operations	13
2.5 Relations, Posets, and Lattices	17
2.6 The Supremum and Infimum of Real Number Sets	20
Exercises	22
Chapter 3: Fuzzy Sets	24
3.1 The Membership Functions of Fuzzy Sets	24
3.2 Inclusion and Operations of Fuzzy Sets	27
3.3 α -Cuts	33
3.4 Convex Fuzzy Sets	36
3.5 Decomposition Theorems	37
3.6 The Extension Principle	40
3.7 Interval Numbers	42
3.8 Fuzzy Numbers and Linguistic Attribute	45
3.9 Binary Operations for Fuzzy Numbers	51
3.10 Fuzzy Integers	58
Exercises	59
Chapter 4: Set Functions	62
4.1 Weights and Classical Measures	63
4.2 Extension of Measures	66
4.3 Monotone Measures	69
4.4 λ -Measures	74

4.5 Quasi-Measures.....	82
4.6 Möbius and Zeta Transformations	87
4.7 Belief Measures and Plausibility Measures	91
4.8 Necessity Measures and Possibility Measures	102
4.9 k -Interactive Measures	107
4.10 Efficiency Measures and Signed Efficiency Measures.....	108
Exercises	112
Chapter 5: Integrations.....	115
5.1 Measurable Functions	115
5.2 The Riemann Integral	123
5.3 The Lebesgue-Like Integral	128
5.4 The Choquet Integral.....	133
5.5 Upper and Lower Integrals.....	153
5.6 r -Integrals on Finite Spaces.....	162
Exercises	174
Chapter 6: Information Fusion.....	177
6.1 Information Sources and Observations.....	177
6.2 Integrals Used as Aggregation Tools	181
6.3 Uncertainty Associated with Set Functions.....	186
6.4 The Inverse Problem of Information Fusion	190
Chapter 7: Optimization and Soft Computing.....	193
7.1 Basic Concepts of Optimization.....	193
7.2 Genetic Algorithms	195
7.3 Pseudo Gradient Search	199
7.4 A Hybrid Search Method	202
Chapter 8: Identification of Set Functions	204
8.1 Identification of λ -Measures	204
8.2 Identification of Belief Measures	206
8.3 Identification of Monotone Measures.....	207
8.3.1 Main algorithm.....	210
8.3.2 Reordering algorithm	211
8.4 Identification of Signed Efficiency Measures by a Genetic Algorithm	213
8.5 Identification of Signed Efficiency Measures by the Pseudo Gradient Search....	215
8.6 Identification of Signed Efficiency Measures Based on the Choquet Integral by an Algebraic Method.....	217
8.7 Identification of Monotone Measures Based on r -Integrals by a Genetic Algorithm.....	219
Chapter 9: Multiregression Based on Nonlinear Integrals	221
9.1 Linear Multiregression	221

9.2 Nonlinear Multiregression Based on the Choquet Integral.....	226
9.3 A Nonlinear Multiregression Model Accommodating Both Categorical and Numerical Predictive Attributes	232
9.4 Advanced Consideration on the Multiregression Involving Nonlinear Integrals	234
9.4.1 Nonlinear multiregressions based on the Choquet integral with quadratic core.....	234
9.4.2 Nonlinear multiregressions based on the Choquet integral involving unknown periodic variation	235
9.4.3 Nonlinear multiregressions based on upper and lower integrals	236
Chapter 10: Classifications Based on Nonlinear Integrals	238
10.1 Classification by an Integral Projection.....	238
10.2 Nonlinear Classification by Weighted Choquet Integrals	242
10.3 An Example of Nonlinear Classification in a Three-Dimensional Sample Space.....	250
10.4 The Uniqueness Problem of the Classification by the Choquet Integral with a Linear Core	263
10.5 Advanced Consideration on the Nonlinear Classification Involving the Choquet Integral	267
10.5.1 Classification by the Choquet integral with the widest gap between classes	267
10.5.2 Classification by cross-oriented projection pursuit	268
10.5.3 Classification by the Choquet integral with quadratic core.....	270
Chapter 11: Data Mining with Fuzzy Data	272
11.1 Defuzzified Choquet Integral with Fuzzy-Valued Integrand (DCIFI).....	273
11.1.1 The α -level set of a fuzzy-valued function.....	274
11.1.2 The Choquet extension of μ	275
11.1.3 Calculation of DCIFI	277
11.2 Classification Model Based on the DCIFI.....	282
11.2.1 Fuzzy data classification by the DCIFI	283
11.2.2 GA-based adaptive classifier-learning algorithm via DCIFI projection pursuit	286
11.2.3 Examples of the classification problems solved by the DCIFI projection classifier.....	290
11.3 Fuzzified Choquet Integral with Fuzzy-Valued Integrand (FCIFI)	300
11.3.1 Definition of the FCIFI	300
11.3.2 The FCIFI with respect to monotone measures.....	303
11.3.3 The FCIFI with respect to signed efficiency measures.....	306
11.3.4 GA-based optimization algorithm for the FCIFI with respect to signed efficiency measures	309

11.4 Regression Model Based on the CIII.....	319
11.4.1 CIII regression model.....	319
11.4.2 Double-GA optimization algorithm	321
11.4.3 Explanatory examples	324
Bibliography	329
Index	337