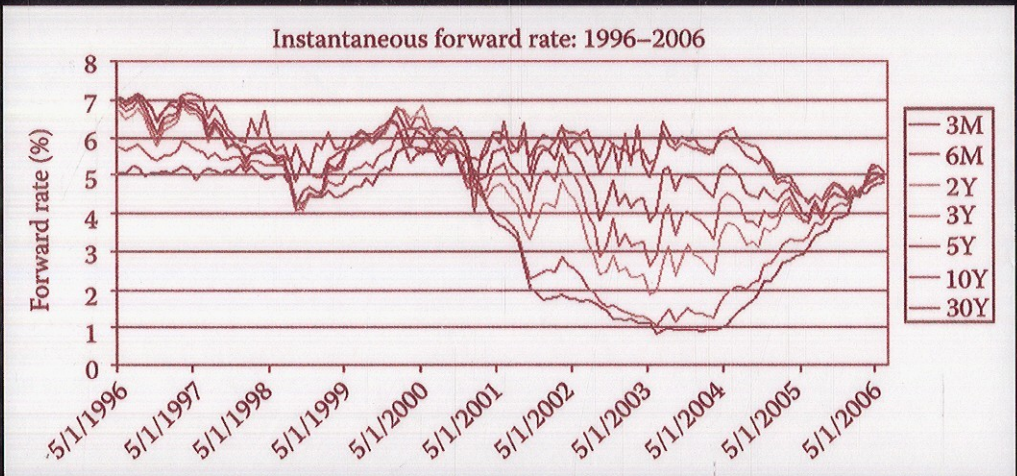


Lixin Wu

# Interest Rate Modeling

## Theory and Practice



Chapman & Hall/CRC FINANCIAL MATHEMATICS SERIES

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