relativity DeMYSTiFieD

A SELF-TEACHING GUIDE

- Take the WARP out of the SPACE/TIME CONTINUUM!
- LEARN a complex subject via HUNDREDS of SIMPLE examples
- Get both the MATH and the PHYSICS of relativity
- Find PRACTICAL TOOLS for problem-solving

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