

Making Everything Easier!™

Quantum Physics Workbook

FOR
DUMMIES®

-  **Review your knowledge and understanding of quantum physics**
-  **Break down equations step by step**
-  **Solve numerous types of quantum physics problems**
-  **Prepare for quizzes and exams**
-  **Point out the tricks instructors use to make problem-solving easier**

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Table of Contents

Introduction	1
About This Book	1
Conventions Used in This Book.....	1
Foolish Assumptions.....	2
How This Book Is Organized	2
Part I: Getting Started with Quantum Physics.....	2
Part II: Round and Round with Angular Momentum and Spin	2
Part III: Quantum Physics in Three Dimensions	2
Part IV: Acting on Impulse — Impacts in Quantum Physics	3
Part V: The Part of Tens.....	3
Icons Used in This Book	3
Where to Go from Here.....	3
 Part 1: Getting Started with Quantum Physics.....	5
 Chapter 1: The Basics of Quantum Physics: Introducing State Vectors	7
Describing the States of a System	7
Becoming a Notation Meister with Bras and Kets.....	12
Getting into the Big Leagues with Operators.....	14
Introducing operators and getting into a healthy, orthonormal relationship	14
Grasping Hermitian operators and adjoints.....	18
Getting Physical Measurements with Expectation Values	18
Commutators: Checking How Different Operators Really Are.....	21
Simplifying Matters by Finding Eigenvectors and Eigenvalues	23
Answers to Problems on State Vectors	27
 Chapter 2: No Handcuffs Involved: Bound States in Energy Wells.	37
Starting with the Wave Function	37
Determining Allowed Energy Levels	40
Putting the Finishing Touches on the Wave Function by Normalizing It	42
Translating to a Symmetric Square Well	44
Banging into the Wall: Step Barriers When the Particle Has Plenty of Energy.....	45
Hitting the Wall: Step Barriers When the Particle Has Doesn't Have Enough Energy	48
Plowing through a Potential Barrier	50
Answers to Problems on Bound States.....	54
 Chapter 3: Over and Over with Harmonic Oscillators	69
Total Energy: Getting On with a Hamiltonian	70
Up and Down: Using Some Crafty Operators	72
Finding the Energy after Using the Raising and Lowering Operators.....	74

Using the Raising and Lowering Operators Directly on the Eigenvectors	76
Finding the Harmonic Oscillator Ground State Wave Function	77
Finding the Excited States' Wave Functions	79
Looking at Harmonic Oscillators in Matrix Terms	82
Answers to Problems on Harmonic Oscillators	85

Part II: Round and Round with Angular Momentum and Spin 95

Chapter 4: Handling Angular Momentum in Quantum Physics97

Rotating Around: Getting All Angular	98
Untangling Things with Commutators	100
Nailing Down the Angular Momentum Eigenvectors	102
Obtaining the Angular Momentum Eigenvalues	104
Scoping Out the Raising and Lowering Operators' Eigenvalues	106
Treating Angular Momentum with Matrices	108
Answers to Problems on Angular Momentum	112

Chapter 5: Spin Makes the Particle Go Round121

Introducing Spin Eigenstates	121
Saying Hello to the Spin Operators: Cousins of Angular Momentum	124
Living in the Matrix: Working with Spin in Terms of Matrices	126
Answers to Problems on Spin Momentum	128

Part III: Quantum Physics in Three Dimensions..... 131

Chapter 6: Solving Problems in Three Dimensions:

Cartesian Coordinates.133

Taking the Schrödinger Equation to Three Dimensions	133
Flying Free with Free Particles in 3-D	136
Getting Physical by Creating Free Wave Packets	138
Getting Stuck in a Box Well Potential.....	141
Box potentials: Finding those energy levels	144
Back to normal: Normalizing the wave function.....	146
Getting in Harmony with 3-D Harmonic Oscillators.....	149
Answers to Problems on 3-D Rectangular Coordinates.....	151

Chapter 7: Going Circular in Three Dimensions: Spherical Coordinates161

Taking It to Three Dimensions with Spherical Coordinates	162
Dealing Freely with Free Particles in Spherical Coordinates	167
Getting the Goods on Spherical Potential Wells.....	170
Bouncing Around with Isotropic Harmonic Oscillators	172
Answers to Problems on 3-D Spherical Coordinates	175

Chapter 8: Getting to Know Hydrogen Atoms183

Eyeing How the Schrödinger Equation Appears for Hydrogen	183
Switching to Center-of-Mass Coordinates to Make the Hydrogen Atom Solvable.....	186

Doing the Splits: Solving the Dual Schrödinger Equation	188
Solving the Radial Schrödinger Equation for $\psi(r)$	190
Juicing Up the Hydrogen Energy Levels	195
Doubling Up on Energy Level Degeneracy	197
Answers to Problems on Hydrogen Atoms	199

Chapter 9: Corraling Many Particles Together207

The 4-1-1 on Many-Particle Systems.....	207
Zap! Working with Multiple-Electron Systems	209
The Old Shell Game: Exchanging Particles.....	211
Examining Symmetric and Antisymmetric Wave Functions	213
Jumping into Systems of Many Distinguishable Particles	215
Trapped in Square Wells: Many Distinguishable Particles	216
Creating the Wave Functions of Symmetric and Antisymmetric Multi-Particle Systems.....	218
Answers to Problems on Multiple-Particle Systems	220

Part IV: Acting on Impulse — Impacts in Quantum Physics 227

Chapter 10: Pushing with Perturbation Theory229

Examining Perturbation Theory with Energy Levels and Wave Functions.....	229
Solving the perturbed Schrödinger equation for the first-order correction	231
Solving the perturbed Schrödinger equation for the second-order correction.....	233
Applying Perturbation Theory to the Real World	235
Answers to Problems on Perturbation Theory.....	237

Chapter 11: One Hits the Other: Scattering Theory245

Cross Sections: Experimenting with Scattering.....	245
A Frame of Mind: Going from the Lab Frame to the Center-of-Mass Frame.....	248
Target Practice: Taking Cross Sections from the Lab Frame to the Center-of-Mass Frame	250
Getting the Goods on Elastic Scattering.....	252
The Born Approximation: Getting the Scattering Amplitude of Particles.....	253
Putting the Born Approximation to the Test	256
Answers to Problems on Scattering Theory	258

Part V: The Part of Tens..... 267

Chapter 12: Ten Tips to Make Solving Quantum Physics Problems Easier. . .269

Normalize Your Wave Functions	269
Use Eigenvalues	269
Meet the Boundary Conditions for Wave Functions	270
Meet the Boundary Conditions for Energy Levels	270
Use Lowering Operators to Find the Ground State	271
Use Raising Operators to Find the Excited States.....	272

Use Tables of Functions.....	273
Decouple the Schrödinger Equation	274
Use Two Schrödinger Equations for Hydrogen	274
Take the Math One Step at a Time	274

Chapter 13: Ten Famous Solved Quantum Physics Problems275

Finding Free Particles.....	275
Enclosing Particles in a Box	275
Grasping the Uncertainty Principle.....	276
Eyeing the Dual Nature of Light and Matter.....	276
Solving for Quantum Harmonic Oscillators	276
Uncovering the Bohr Model of the Atom.....	276
Tunneling in Quantum Physics.....	277
Understanding Scattering Theory	277
Deciphering the Photoelectric Effect.....	277
Unraveling the Spin of Electrons	277

Chapter 14: Ten Ways to Avoid Common Errors When Solving Problems. . . .279

Translate between Kets and Wave Functions.....	279
Take the Complex Conjugate of Operators	279
Take the Complex Conjugate of Wave Functions	280
Include the Minus Sign in the Schrödinger Equation.....	280
Include $\sin \theta$ in the Laplacian in Spherical Coordinates	280
Remember that $\lambda \ll 1$ in Perturbation Hamiltonians	281
Don't Double Up on Integrals.....	281
Use a Minus Sign for Antisymmetric Wave Functions under Particle Exchange....	281
Remember What a Commutator Is	282
Take the Expectation Value When You Want Physical Measurements.....	282