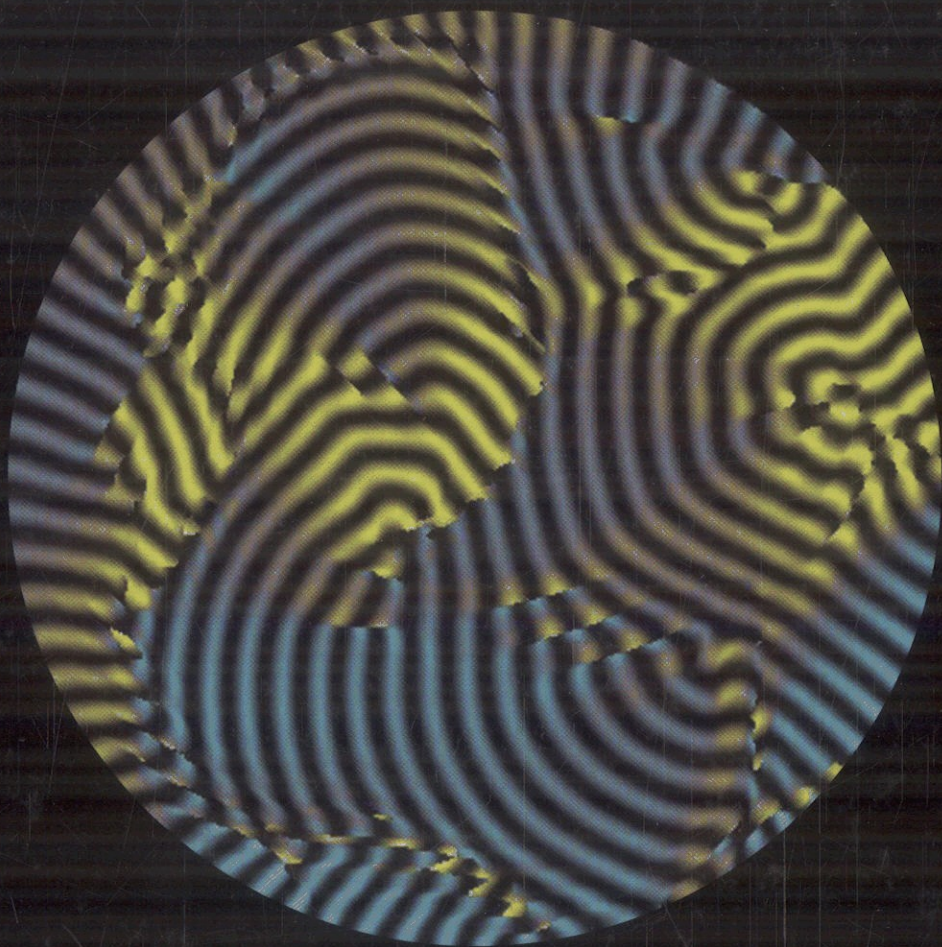


Pattern Formation and Dynamics in Nonequilibrium Systems



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CAMBRIDGE

Contents

| | |
|--|------------------|
| <i>Preface</i> | <i>page</i> xiii |
| 1 Introduction | 1 |
| 1.1 The big picture: why is the Universe not boring? | 2 |
| 1.2 Convection: a first example of a nonequilibrium system | 3 |
| 1.3 Examples of nonequilibrium patterns and dynamics | 10 |
| 1.3.1 Natural patterns | 10 |
| 1.3.2 Prepared patterns | 20 |
| 1.3.3 What are the interesting questions? | 35 |
| 1.4 New features of pattern-forming systems | 38 |
| 1.4.1 Conceptual differences | 38 |
| 1.4.2 New <i>properties</i> | 43 |
| 1.5 A strategy for studying pattern-forming nonequilibrium systems | 44 |
| 1.6 Nonequilibrium systems not discussed in this book | 48 |
| 1.7 Conclusion | 49 |
| 1.8 Further reading | 50 |
| 2 Linear instability: basics | 56 |
| 2.1 Conceptual framework for a linear stability analysis | 57 |
| 2.2 Linear stability analysis of a pattern-forming system | 63 |
| 2.2.1 One-dimensional Swift–Hohenberg equation | 63 |
| 2.2.2 Linear stability analysis | 64 |
| 2.2.3 Growth rates and instability diagram | 67 |
| 2.3 Key steps of a linear stability analysis | 69 |
| 2.4 Experimental investigations of linear stability | 70 |
| 2.4.1 General remarks | 70 |
| 2.4.2 Taylor–Couette instability | 74 |

| | | |
|----------|--|------------|
| 2.5 | Classification for linear instabilities of a uniform state | 75 |
| 2.5.1 | Type-I instability | 77 |
| 2.5.2 | Type-II instability | 79 |
| 2.5.3 | Type-III instability | 80 |
| 2.6 | Role of symmetry in a linear stability analysis | 81 |
| 2.6.1 | Rotationally invariant systems | 82 |
| 2.6.2 | Uniaxial systems | 84 |
| 2.6.3 | Anisotropic systems | 86 |
| 2.6.4 | Formal discussion | 86 |
| 2.7 | Conclusions | 88 |
| 2.8 | Further reading | 88 |
| 3 | Linear instability: applications | 96 |
| 3.1 | Turing instability | 96 |
| 3.1.1 | Reaction–diffusion equations | 97 |
| 3.1.2 | <i>Linear stability analysis</i> | 99 |
| 3.1.3 | Oscillatory instability | 108 |
| 3.2 | Realistic chemical systems | 109 |
| 3.2.1 | Experimental apparatus | 109 |
| 3.2.2 | Evolution equations | 110 |
| 3.2.3 | Experimental results | 116 |
| 3.3 | Conclusions | 119 |
| 3.4 | Further reading | 120 |
| 4 | Nonlinear states | 126 |
| 4.1 | Nonlinear saturation | 129 |
| 4.1.1 | Complex amplitude | 130 |
| 4.1.2 | Bifurcation theory | 134 |
| 4.1.3 | Nonlinear stripe state of the Swift–Hohenberg equation | 137 |
| 4.2 | Stability balloons | 139 |
| 4.2.1 | General discussion | 139 |
| 4.2.2 | Busse balloon for Rayleigh–Bénard convection | 147 |
| 4.3 | Two-dimensional lattice states | 152 |
| 4.4 | Non-ideal states | 158 |
| 4.4.1 | Realistic patterns | 158 |
| 4.4.2 | Topological defects | 160 |
| 4.4.3 | Dynamics of defects | 164 |
| 4.5 | Conclusions | 165 |
| 4.6 | Further reading | 166 |

| | | |
|----------|---|------------|
| 5 | Models | 173 |
| 5.1 | Swift–Hohenberg model | 175 |
| 5.1.1 | Heuristic derivation | 176 |
| 5.1.2 | Properties | 179 |
| 5.1.3 | Numerical simulations | 183 |
| 5.1.4 | Comparison with experimental systems | 185 |
| 5.2 | Generalized Swift–Hohenberg models | 187 |
| 5.2.1 | Non-symmetric model | 187 |
| 5.2.2 | Nonpotential models | 188 |
| 5.2.3 | Models with mean flow | 188 |
| 5.2.4 | Model for rotating convection | 190 |
| 5.2.5 | Model for quasicrystalline patterns | 192 |
| 5.3 | Order-parameter equations | 192 |
| 5.4 | Complex Ginzburg–Landau equation | 196 |
| 5.5 | Kuramoto–Sivashinsky equation | 197 |
| 5.6 | Reaction–diffusion models | 199 |
| 5.7 | Models that are discrete in space, time, or value | 201 |
| 5.8 | Conclusions | 201 |
| 5.9 | Further reading | 202 |
| 6 | One-dimensional amplitude equation | 208 |
| 6.1 | Origin and meaning of the amplitude | 211 |
| 6.2 | Derivation of the amplitude equation | 214 |
| 6.2.1 | Phenomenological derivation | 214 |
| 6.2.2 | Deduction of the amplitude-equation parameters | 217 |
| 6.2.3 | Method of multiple scales | 218 |
| 6.2.4 | Boundary conditions for the amplitude equation | 219 |
| 6.3 | Properties of the amplitude equation | 221 |
| 6.3.1 | Universality and scales | 221 |
| 6.3.2 | Potential dynamics | 224 |
| 6.4 | Applications of the amplitude equation | 226 |
| 6.4.1 | Lateral boundaries | 226 |
| 6.4.2 | Eckhaus instability | 230 |
| 6.4.3 | Phase dynamics | 234 |
| 6.5 | Limitations of the amplitude-equation formalism | 237 |
| 6.6 | Conclusions | 238 |
| 6.7 | Further reading | 239 |
| 7 | Amplitude equations for two-dimensional patterns | 244 |
| 7.1 | Stripes in rotationally invariant systems | 246 |
| 7.1.1 | Amplitude equation | 246 |
| 7.1.2 | Boundary conditions | 248 |

| | | |
|----------|---|------------|
| 7.1.3 | Potential | 249 |
| 7.1.4 | Stability balloon | 250 |
| 7.1.5 | Phase dynamics | 252 |
| 7.2 | Stripes in anisotropic systems | 253 |
| 7.2.1 | Amplitude equation | 253 |
| 7.2.2 | Stability balloon | 254 |
| 7.2.3 | Phase dynamics | 255 |
| 7.3 | Superimposed stripes | 255 |
| 7.3.1 | Amplitude equations | 256 |
| 7.3.2 | Competition between stripes and lattices | 261 |
| 7.3.3 | Hexagons in the absence of field-inversion symmetry | 264 |
| 7.3.4 | Spatial variations | 269 |
| 7.3.5 | Cross-stripe instability | 270 |
| 7.4 | Conclusions | 272 |
| 7.5 | Further reading | 273 |
| 8 | Defects and fronts | 279 |
| 8.1 | Dislocations | 281 |
| 8.1.1 | Stationary dislocation | 283 |
| 8.1.2 | Dislocation dynamics | 285 |
| 8.1.3 | Interaction of dislocations | 289 |
| 8.2 | Grain boundaries | 290 |
| 8.3 | Fronts | 296 |
| 8.3.1 | Existence of front solutions | 296 |
| 8.3.2 | Front selection | 303 |
| 8.3.3 | Wave-number selection | 307 |
| 8.4 | Conclusions | 309 |
| 8.5 | Further reading | 309 |
| 9 | Patterns far from threshold | 315 |
| 9.1 | Stripe and lattice states | 317 |
| 9.1.1 | Goldstone modes and phase dynamics | 318 |
| 9.1.2 | Phase diffusion equation | 320 |
| 9.1.3 | Beyond the phase equation | 327 |
| 9.1.4 | Wave-number selection | 331 |
| 9.2 | Novel patterns | 337 |
| 9.2.1 | Pinning and disorder | 338 |
| 9.2.2 | Localized structures | 340 |
| 9.2.3 | Patterns based on front properties | 342 |
| 9.2.4 | Spatiotemporal chaos | 345 |
| 9.3 | Conclusions | 352 |
| 9.4 | Further reading | 353 |

| | | |
|-----------|--|------------|
| 10 | Oscillatory patterns | 358 |
| 10.1 | Convective and absolute instability | 360 |
| 10.2 | States arising from a type-III-o instability | 363 |
| 10.2.1 | Phenomenology | 363 |
| 10.2.2 | Amplitude equation | 365 |
| 10.2.3 | Phase equation | 368 |
| 10.2.4 | Stability balloon | 370 |
| 10.2.5 | Defects: sources, sinks, shocks, and spirals | 372 |
| 10.3 | Unidirectional waves in a type-I-o system | 379 |
| 10.3.1 | Amplitude equation | 380 |
| 10.3.2 | Criterion for absolute instability | 382 |
| 10.3.3 | Absorbing boundaries | 383 |
| 10.3.4 | Noise-sustained structures | 384 |
| 10.3.5 | Local modes | 386 |
| 10.4 | Bidirectional waves in a type-I-o system | 388 |
| 10.4.1 | Traveling and standing waves | 389 |
| 10.4.2 | Onset in finite geometries | 390 |
| 10.4.3 | Nonlinear waves with reflecting boundaries | 392 |
| 10.5 | Waves in a two-dimensional type-I-o system | 393 |
| 10.6 | Conclusions | 395 |
| 10.7 | Further reading | 396 |
| 11 | Excitable media | 401 |
| 11.1 | Nerve fibers and heart muscle | 404 |
| 11.1.1 | Hodgkin–Huxley model of action potentials | 404 |
| 11.1.2 | Models of electrical signaling in the heart | 411 |
| 11.1.3 | FitzHugh–Nagumo model | 413 |
| 11.2 | Oscillatory or excitable | 416 |
| 11.2.1 | Relaxation oscillations | 419 |
| 11.2.2 | Excitable dynamics | 420 |
| 11.3 | Front propagation | 421 |
| 11.4 | Pulses | 424 |
| 11.5 | Waves | 426 |
| 11.6 | Spirals | 430 |
| 11.6.1 | Structure | 430 |
| 11.6.2 | Formation | 436 |
| 11.6.3 | Instabilities | 437 |
| 11.6.4 | Three dimensions | 439 |
| 11.6.5 | Application to heart arrhythmias | 439 |
| 11.7 | Further reading | 441 |

| | |
|--|------------|
| 12 Numerical methods | 445 |
| 12.1 Introduction | 445 |
| 12.2 Discretization of fields and equations | 447 |
| 12.2.1 Finitely many operations on a finite amount of data | 447 |
| 12.2.2 The discretization of continuous fields | 449 |
| 12.2.3 The discretization of equations | 451 |
| 12.3 Time integration methods for pattern-forming systems | 457 |
| 12.3.1 Overview | 457 |
| 12.3.2 Explicit methods | 460 |
| 12.3.3 Implicit methods | 465 |
| 12.3.4 Operator splitting | 470 |
| 12.3.5 How to choose the spatial and temporal resolutions | 473 |
| 12.4 Stationary states of a pattern-forming system | 475 |
| 12.4.1 Iterative methods | 476 |
| 12.4.2 Newton's method | 477 |
| 12.5 Conclusion | 482 |
| 12.6 Further reading | 485 |
| <i>Appendix 1 Elementary bifurcation theory</i> | 496 |
| <i>Appendix 2 Multiple-scales perturbation theory</i> | 503 |
| <i>Glossary</i> | 520 |
| <i>References</i> | 526 |
| <i>Index</i> | 531 |