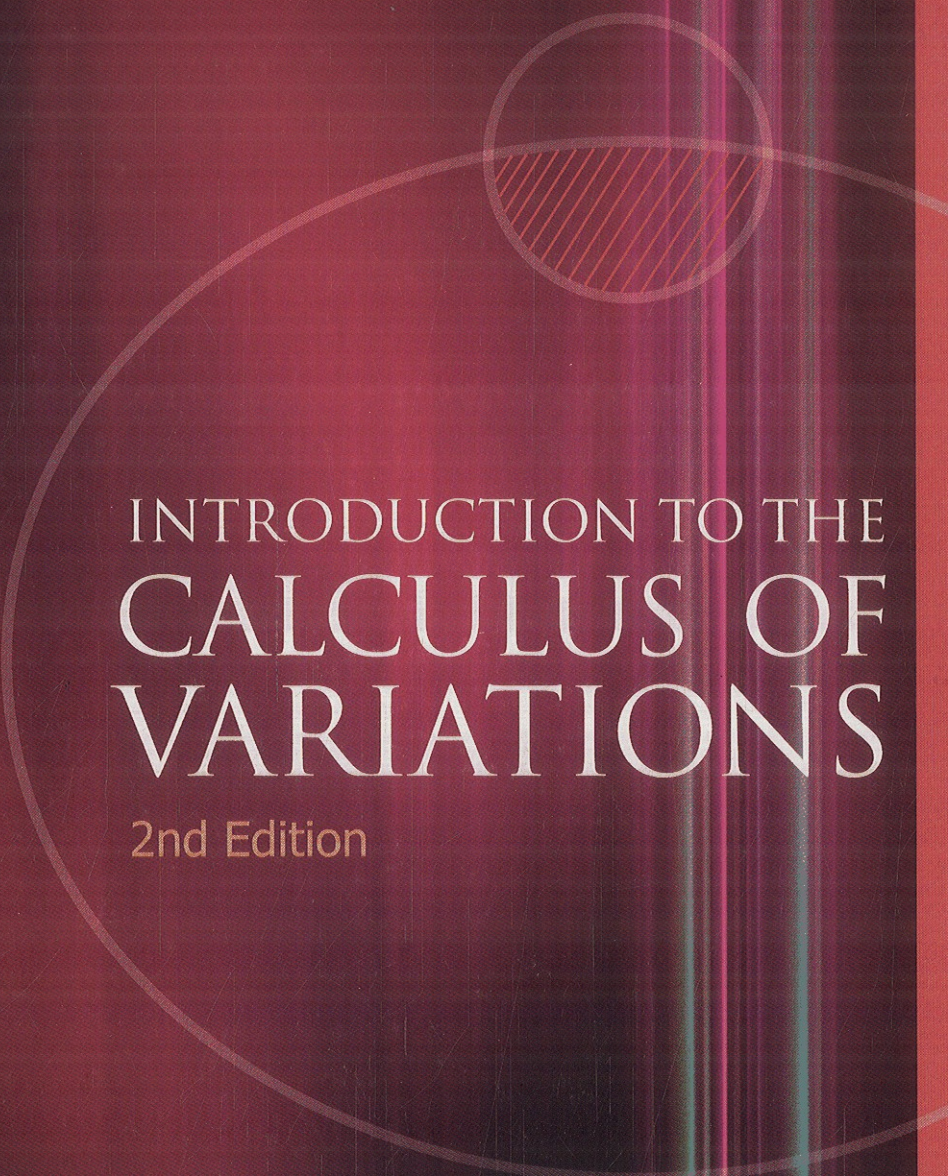


BERNARD DACOROGNA



INTRODUCTION TO THE  
CALCULUS OF  
VARIATIONS

2nd Edition

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