

**ELECTRONIC
ENGINEERING**

AIDED NAVIGATION

**GPS with High Rate
Sensors**

JAY A. FARRELL



Contents

I	Theory	1
	Part I Overview	3
1	Introduction	5
1.1	Method Overview	7
1.1.1	Methodology Example	8
1.1.2	Methodology Summary	10
1.2	Overview of Part I: Theory	12
1.2.1	Reference Frames	12
1.2.2	Deterministic Systems	13
1.2.3	Stochastic Processes	13
1.2.4	Optimal State Estimation	14
1.2.5	Performance Analysis	15
1.2.6	Aided Navigation System Design and Analysis	15
1.3	Overview of Part II: Applications	16
1.3.1	GPS	16
1.3.2	Aided Navigation Systems	16
1.4	Overview of Appendices	17
2	Reference Frames	19
2.1	Reference Frame Properties	21
2.2	Reference Frame Definitions	23
2.2.1	Inertial Frame	23
2.2.2	Earth Centered Earth Fixed (ECEF) Frames	24
2.2.3	Geographic Frame	24
2.2.4	Geocentric Frame	24
2.2.5	Local Geodetic or Tangent Plane	25
2.2.6	Body Frame	26
2.2.7	Platform Frame	27
2.2.8	Instrument Frames	27
2.2.9	Summary	27
2.3	ECEF Coordinate Systems	28

2.3.1	ECEF Rectangular Coordinates	29
2.3.2	The Earth Geoid and Gravity Model	29
2.3.3	ECEF Transformations	33
2.4	Reference Frame Transformations	35
2.4.1	The Direction Cosine Matrix	35
2.4.2	Point Transformation	39
2.4.3	Vector Transformation	39
2.4.4	Matrix Transformation	40
2.5	Specific Vector Transformations	41
2.5.1	Plane Rotations	41
2.5.2	Transformation: ECEF to Tangent Plane	42
2.5.3	Transformation: ECEF to Geographic	44
2.5.4	Transformation: Vehicle to Navigation Frame	46
2.5.5	Transformation: Orthogonal Small Angle	50
2.6	Rotating Reference Frames	51
2.6.1	Direction Cosine Kinematics	51
2.6.2	Derivative Calculations in Rotation Frames	53
2.7	Calculation of the Direction Cosine	54
2.7.1	Direction Cosine Derivatives	55
2.7.2	Euler Angle Derivatives	56
2.8	References and Further Reading	58
2.9	Exercises	58
3	Deterministic Systems	63
3.1	Continuous-Time Systems Models	63
3.1.1	Ordinary Differential Equations	64
3.1.2	Transfer Functions	65
3.1.3	State Space	66
3.2	State Augmentation	69
3.3	State Space Linearization	72
3.4	Discrete-Time State Space Notation	74
3.5	State Space Analysis	74
3.5.1	Similarity Transformation	75
3.5.2	State Space to Transfer Function	76
3.5.3	State Transition Matrix Properties	79
3.5.4	Linear Time-Invariant Systems	80
3.5.5	Discrete-Time Equivalent Models	81
3.6	State Estimation	82
3.6.1	Observability	85
3.6.2	Estimator Design by Pole Placement	87
3.6.3	Observable Subspace	92
3.7	References and Further Reading	95
3.8	Exercises	95

4	Stochastic Processes	105
4.1	Basic Stochastic Process Concepts	105
4.1.1	Examples	106
4.1.2	Plan of Study	110
4.2	Scalar Random Variables	110
4.2.1	Basic Properties	110
4.2.2	Gaussian Distributions	112
4.2.3	Transformations of Scalar Random Variables	113
4.3	Multiple Random Variables	115
4.3.1	Basic Properties	115
4.3.2	Statistics and Statistical Properties	117
4.3.3	Vector Gaussian Random Variables	120
4.3.4	Transformations of Vector Random Variables	120
4.4	Stochastic Processes	121
4.4.1	Statistics and Statistical Properties	121
4.4.2	White and Colored Noise	123
4.5	Linear Systems with Random Inputs	125
4.6	State Models for Stochastic Processes	130
4.6.1	Standard Model	131
4.6.2	Stochastic Systems and State Augmentation	132
4.6.3	Gauss-Markov Processes	133
4.6.4	Time-propagation of the Mean	139
4.6.5	Time-propagation of the Variance	139
4.7	Discrete-time Equivalent Models	140
4.7.1	Calculation of Φ_k from $\mathbf{F}(t)$	140
4.7.2	Calculation of \mathbf{Qd}_k from $\mathbf{Q}(t)$	141
4.8	Linear State Estimation	144
4.9	Detailed Examples	146
4.9.1	System Performance Metrics	146
4.9.2	Instrument Specifications	154
4.9.3	One Dimensional INS	157
4.9.4	One Dimensional Position Aided INS	159
4.10	Complementary Filtering	161
4.11	References and Further Reading	162
4.12	Exercises	163
5	Optimal State Estimation	169
5.1	State Estimation: Review	170
5.2	Minimum Variance Gain Derivation	172
5.2.1	Kalman Gain Derivation	172
5.2.2	Kalman Gain: Posterior Covariance	173
5.2.3	Summary	173
5.3	From WLS to the Kalman Filter	174
5.3.1	Weighted Least Squares (WLS)	174

5.3.2	Weighted Least Squares Solution	175
5.3.3	Recursive Least Squares (RLS)	179
5.3.4	Kalman Filtering	183
5.4	Kalman Filter Derivation Summary	184
5.4.1	Equivalent Measurement Updates	186
5.4.2	Equivalent Covariance Measurement Updates	187
5.4.3	Kalman Filter Examples	187
5.5	Kalman Filter Properties	189
5.6	Implementation Issues	191
5.6.1	Scalar Measurement Processing	191
5.6.2	Correlated Measurements	193
5.6.3	Bad or Missing Data	194
5.7	Implementation Sequence	195
5.8	Asynchronous Measurements	195
5.9	Numeric Issues	196
5.9.1	Covariance Matrix Symmetry	196
5.9.2	Covariance Matrix Positive Definiteness	197
5.10	Suboptimal Filtering	197
5.10.1	Deleting States	198
5.10.2	Schmidt-Kalman Filtering	199
5.10.3	Decoupling	202
5.10.4	Off-line Gain Calculation	202
5.10.5	Nonlinear Filtering	203
5.11	References and Further Reading	211
5.12	Exercises	211
6	Performance Analysis	217
6.1	Covariance Analysis	217
6.2	Monte Carlo Analysis	224
6.3	Error Budgeting	224
6.4	Covariance Divergence	230
6.5	References and Further Reading	232
6.6	Exercise	233
7	Navigation System Design	235
7.1	Methodology Summary	235
7.2	Methodology: Detailed Example	237
7.2.1	Augmented Kinematic Model	238
7.2.2	Navigation Mechanization Equations	238
7.2.3	Sensor Models	239
7.2.4	Error Models	240
7.2.5	State Estimator Design	240
7.2.6	Covariance Analysis	244
7.3	Complementary Filtering	247

CONTENTS

7.3.1	Frequency Domain Approach	248
7.3.2	Kalman Filter Approach	250
7.4	An Alternative Approach	251
7.4.1	Total State: Kinematic Model	252
7.4.2	Total State: Time Update	252
7.4.3	Total State: Measurement Update	253
7.5	Approach Comparison	254
7.6	A Caution	255
7.7	References and Further Reading	256
7.8	Exercises	256

II Application 259

Part II Overview 261

8 Global Positioning System 263

8.1	GPS Overview	264
8.1.1	GPS System	264
8.1.2	Original GPS Signal	265
8.2	GPS Pseudorange	266
8.2.1	GPS Pseudorange Notation	267
8.2.2	GPS Pseudorange Solution	269
8.2.3	Satellite Azimuth and Elevation	273
8.3	GPS Receiver Overview	276
8.3.1	Carrier Phase Observables	277
8.3.2	Delta Pseudorange Observable	278
8.4	GPS URE Characteristics	280
8.4.1	Clocks	281
8.4.2	Satellite Clock Bias, $c\delta t^s$	282
8.4.3	Receiver Clock Error, $\Delta\tau_r$	283
8.4.4	Atmospheric Delay, $c\delta t_a^s$	287
8.4.5	Ephemeris Errors, E^s	292
8.4.6	Selective Availability, SA^s	293
8.4.7	Multipath, M_ρ^s, M_ϕ^s	294
8.4.8	Receiver Noise, η_ρ^i, η_ϕ^i	295
8.4.9	Carrier Tracking and Integer Ambiguity, N^i	295
8.4.10	Summary	301
8.5	Geometric Dilution of Precision	302
8.6	Two Frequency Receivers	306
8.6.1	Wide and Narrow Lane Observables	309
8.7	Carrier-Smoothed Code	310
8.8	Differential GPS	312
8.8.1	Relative DGPS	313

8.8.2	Differential GPS	317
8.8.3	Double Differences	323
8.9	Integer Ambiguity Resolution	325
8.9.1	Decreasing the Search Space	328
8.9.2	Selection of Optimal Integers	329
8.9.3	Modernized GPS Signal	331
8.10	GPS Summary	332
8.11	References and Further Reading	333
9	GPS Aided Encoder-Based Dead-Reckoning	335
9.1	Encoder Model	336
9.2	Kinematic Model	338
9.3	Encoder Navigation Equations	340
9.3.1	Continuous-Time: Theory	340
9.3.2	Discrete-Time: Implementation	341
9.4	Error State Dynamic Model	341
9.5	GPS Aiding	342
9.5.1	Receiver Clock Modeling	343
9.5.2	Measurement Differencing	344
9.5.3	Comparison	345
9.6	Performance Analysis	346
9.6.1	Observability	346
9.6.2	Covariance Analysis	348
9.7	General 3-d Problem	351
10	AHRS	353
10.1	Kinematic Model	354
10.2	Sensor Models	355
10.3	Initialization	356
10.3.1	State Initialization: Approach 1	356
10.3.2	State Initialization: Approach 2	358
10.4	AHRS Mechanization Equations	358
10.5	Error Models	359
10.5.1	Measurement Error Model	360
10.5.2	Attitude Error Dynamics	363
10.5.3	AHRS State Space Error Model	365
10.5.4	Measurement Noise Covariance	366
10.5.5	Initial Error Covariance Matrix	367
10.6	AHRS Approach Summary	368
10.7	Observability and Performance Analysis	369
10.8	Pitch and Roll Application	370
10.9	References and Further Reading	377

11 Aided Inertial Navigation	379
11.1 Gravitation and Specific Force	379
11.1.1 Gravitation	379
11.1.2 Specific Force	380
11.1.3 Accelerometers	381
11.1.4 Gravity Error	384
11.2 INS Kinematic Equations	386
11.2.1 Inertial Frame	388
11.2.2 ECEF Frame	388
11.2.3 Tangent Frame	389
11.2.4 Geographic Frame	389
11.3 INS Mechanization Equations	390
11.4 INS Error State Dynamic Equations	392
11.4.1 Position Error Linearization	393
11.4.2 Attitude Error Linearization	393
11.4.3 Velocity Error Linearization	395
11.5 INS Error Characteristics	396
11.5.1 Simplified Error Models	397
11.5.2 Full Error Model	399
11.6 Augmented State Equations	406
11.6.1 Instrument Error Overview	407
11.6.2 Accelerometer Error Modeling	408
11.6.3 Gyro Error Modeling	411
11.6.4 Error Characteristics	414
11.7 Initialization	414
11.7.1 Self-Alignment Techniques	416
11.8 Aiding Measurements	421
11.8.1 Position Aiding	421
11.8.2 GPS Pseudorange Aiding	422
11.9 Observability Analysis	427
11.9.1 Stationary, Level, Known Biases	428
11.9.2 Stationary, Level, Unknown Biases	429
11.10References and Further Reading	430
12 LBL and Doppler Aided INS	431
12.1 Kinematics	432
12.1.1 Notation	432
12.1.2 System Kinematics	433
12.2 Sensors	433
12.2.1 Inertial Measurement Unit	434
12.2.2 Attitude and Yaw Sensor	435
12.2.3 Doppler Velocity Log	436
12.2.4 Pressure Sensor	436
12.2.5 Long Baseline Transceiver	436

12.3	Mechanization and IMU Processing	438
12.3.1	Mechanization Equations	438
12.3.2	IMU Processing	438
12.4	Error State Dynamic Model	439
12.4.1	Position Error Model	439
12.4.2	Velocity Error Model	440
12.4.3	Attitude Error Model	441
12.4.4	Calibration Parameter Error Models	442
12.4.5	Error Model Summary	442
12.5	Aiding Measurement Models	443
12.5.1	Attitude and Yaw Prediction	443
12.5.2	Doppler Prediction	444
12.5.3	Depth Prediction	445
12.5.4	LBL Prediction	445
12.6	EKF Sensor Integration	446
12.6.1	Measurement Updates for $t \in [t_0, t_4]$	447
12.6.2	Measurement Updates for $t \notin [t_0, t_4]$	448
12.6.3	Covariance Propagation	449
12.7	Observability	449
12.8	Simulation Performance	452
A	Notation	455
A.1	Notation	455
A.2	Useful Constants	457
A.3	Acronyms	457
A.4	Other Notation	458
B	Linear Algebra Review	459
B.1	Definitions	459
B.2	Matrix and Vector Operations	461
B.3	Independence and Determinants	464
B.4	Matrix Inversion	465
B.5	Matrix Inversion Lemma	466
B.6	Eigenvalues and Eigenvectors	467
B.7	Positive Definite Matrices	468
B.8	Singular Value Decomposition	469
B.9	Orthogonalization	469
B.10	LU Decomposition	470
B.11	UD Decomposition	472
B.12	Matrix Exponential	472
B.12.1	Power Series	472
B.12.2	Laplace Transform	473
B.13	Matrix Calculus	474
B.13.1	Derivatives with Respect to Scalars	474

CONTENTS

B.13.2 Derivatives with Respect to Vectors	474
B.13.3 Derivatives with Respect to Matrices	475
B.14 Numeric Zero Finding & Optimization	475
B.14.1 Numerical Zero Finding	475
B.14.2 Numeric Optimization	477
B.15 References and Further Reading	480
B.16 Exercises	480
C Calculation of GPS Satellite Position & Velocity	485
C.1 Satellite Clock Corrections	487
C.2 Satellite Position Calculations	488
C.3 Reference Frame Consistency	494
C.4 Satellite Velocity	495
C.4.1 Equations from Ephemeris	495
C.4.2 Practical Issues	496
C.5 Ionospheric Model	497
C.6 References and Further Reading	500
D Quaternions	501
D.1 Quaternions Basics	501
D.2 Rotations	503
D.2.1 Direction Cosine to Quaternion	504
D.2.2 <i>Quaternions to Euler Angles</i>	505
D.3 Quaternion Derivative	505
D.3.1 General Derivation	505
D.3.2 Body to Navigation Frame Result	506
D.4 Summary	507
D.5 Quaternion Integration	508
D.6 Attitude Representation Comparison	510
D.7 References and Further Reading	511
D.8 Exercises	511
Bibliography	515
Index	527