

BIRKHÄUSER

APPLIED AND NUMERICAL HARMONIC ANALYSIS

Four Short Courses on Harmonic Analysis

Wavelets, Frames,
Time-Frequency Methods,
and Applications to
Signal and Image Analysis

Brigitte Forster
Peter Massopust

Editors

With Contributions by
Ole Christensen
Karlheinz Gröchenig
Demetrio Labate
Pierre Vandergheynst
Guido Weiss
Yves Wiaux

Contents

ANHA Series Preface	v
Preface	ix
List of Contributors	xvii
1 Introduction: Mathematical Aspects of Time-Frequency Analysis ...	1
Peter Massopust and Brigitte Forster	
1.1 Aims of Time-Frequency Analysis	1
1.1.1 Signal and Model	2
1.1.2 Transforms	3
1.1.3 Signal Manipulations—Filters	8
1.1.4 Why Discretizing? Techniques, Challenges, Pitfalls	8
1.2 Basic Methods of Time-Frequency Analysis: Orthonormal Bases and Generalized Fourier Series	10
1.2.1 Schauder Bases in Banach Spaces	10
1.2.2 Generalized Fourier Series	14
1.3 The Fourier Integral Transform	15
1.3.1 Definition and Properties	15
1.3.2 The Plancherel Transform	26
1.3.3 The Theorem of Paley–Wiener	29
1.3.4 Discretization: The Poisson Summation Formula and the Sampling Theorem	30
1.4 Windowed Fourier Transforms	31
1.4.1 The Short-Time Fourier Transform (STFT)	31
1.4.2 The Gabor Transform	32
1.4.3 The Heisenberg Uncertainty Principle	36
1.4.4 Discretization: Gabor Frames	38
1.4.5 Shortcomings of the Windowed Fourier Transform	38
1.5 The Wavelet Transform	39
1.5.1 Definition and Properties	39
1.5.2 Scale Discretization—The Dyadic Wavelet Transform ...	42
1.5.3 Multiresolution Analyses	43
1.6 Other Multiscale Transforms	45
1.6.1 Tensor Product Wavelets in 2D	45
1.6.2 Some Wavelet-Type Transforms	46
1.6.3 Moving to Other Manifolds—Wavelets on the Sphere ...	47
Exercises	49

2	B-Spline Generated Frames	51
	Ole Christensen	
2.1	Introduction	51
2.2	Bessel Sequences in Hilbert Spaces	52
2.3	General Bases and Orthonormal Bases	54
2.4	Riesz Bases	56
2.5	Frames and Their Properties	59
2.6	Frames and Riesz Bases	62
2.7	B-Splines	64
2.8	Frames of Translates	65
2.9	Basic Gabor Frame Theory	67
2.10	Tight Gabor Frames	70
2.11	The Duals of a Gabor Frame	72
2.12	Explicit Construction of Dual Gabor Frame Pairs	73
2.13	Wavelets and the Unitary Extension Principle	77
	Exercises	84
3	Continuous and Discrete Reproducing Systems That Arise from Translations. Theory and Applications of Composite Wavelets	87
	Demetrio Labate and Guido Weiss	
3.1	Introduction	87
3.2	Unified Theory of Reproducing Systems	91
3.2.1	Unified Theorem for Reproducing Systems	92
3.3	Continuous Wavelet Transform	98
3.3.1	Admissible Groups	101
3.3.2	Wave Packet Systems	102
3.4	Affine Systems with Composite Dilations	104
3.4.1	Affine System with Composite Dilations	108
3.4.2	Other Examples	110
3.5	Continuous Shearlet Transform	116
3.5.1	Edge Analysis Using the Shearlet Transform	120
3.5.2	A Shearlet Approach to Edge Analysis and Detection	121
3.5.3	Discrete Shearlet System	123
3.5.4	Optimal Representations Using Shearlets	126
	Exercises	129
4	Wavelets on the Sphere	131
	Pierre Vandergheynst and Yves Wiaux	
4.1	Introduction	131
4.2	Scale-Space Premises	132
4.2.1	Directional Correlations	132
4.2.2	Harmonic Analysis	133
4.2.3	Affine Transformations	136
4.3	Continuous Formalism	141
4.3.1	Generic Wavelets	141
4.3.2	Stereographic Wavelets	145

4.3.3	Kernel Wavelets	149
4.3.4	Discretization of Variables	152
4.4	Analysis Algorithms	153
4.4.1	Pixelization	153
4.4.2	Fast Algorithms	155
4.5	Discrete Formalism	159
4.5.1	Discrete Wavelets	159
4.5.2	Other Constructions	165
4.6	Reconstruction Algorithm	167
4.6.1	Multiresolution	167
4.6.2	Fast Algorithm	168
4.7	Applications	169
4.7.1	Cosmic Microwave Background Analysis	169
4.7.2	Human Cortex Image Denoising	170
4.8	Conclusion	173
	Exercises	174
5	Wiener's Lemma: Theme and Variations. An Introduction to Spectral Invariance and Its Applications	175
	Karlheinz Gröchenig	
5.1	Introduction	175
5.2	Wiener's Lemma—Classical	177
5.2.1	Definitions from Banach Algebras	178
5.2.2	Absolutely Convergent Fourier Series	178
5.2.3	Wiener's Lemma	179
5.2.4	Proof of Wiener's Lemma	180
5.2.5	Abstract Concepts—Inverse-Closedness	182
5.2.6	Convolution Operators	188
	Exercises for Section 5.2	193
5.3	Variations	195
5.3.1	Weighted Versions of Wiener's Lemma	195
5.3.2	Matrix Algebras	200
5.3.3	Absolutely Convergent Series of Time-Frequency Shifts	208
5.3.4	Convolution Operators on Groups	216
5.3.5	Pseudodifferential Operators	220
5.3.6	Time-Varying Systems and Wireless Communications	227
	Exercises for Section 5.3	234
	References	235
	Index	245