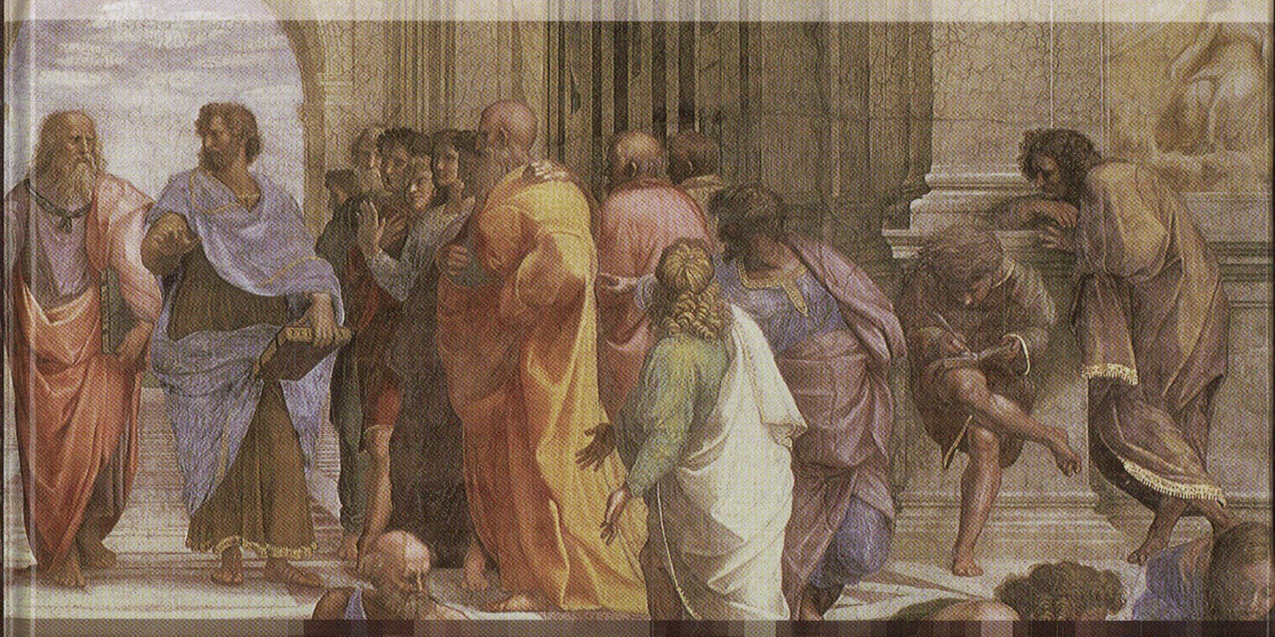
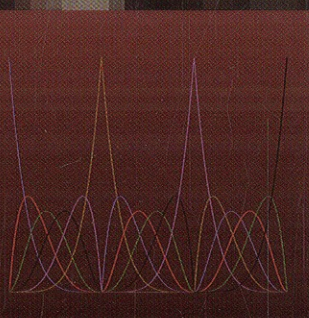
Isogeometric Analysis

Toward Integration of CAD and FEA



J. Austin Cottrell Thomas J. R. Hughes Yuri Bazilevs



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