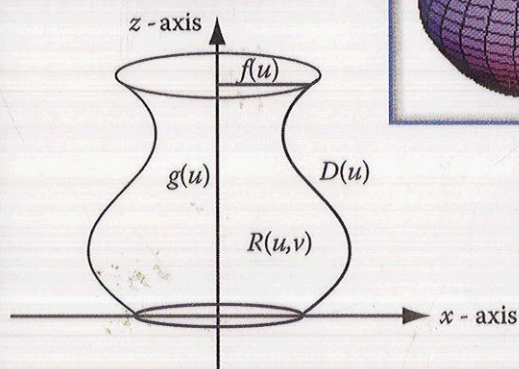
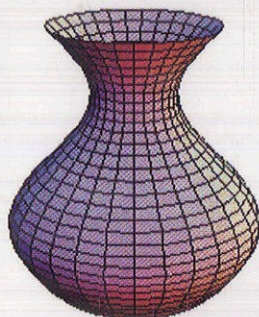
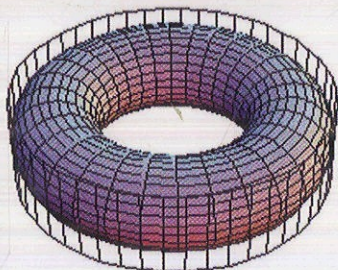
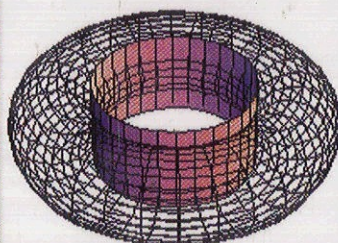
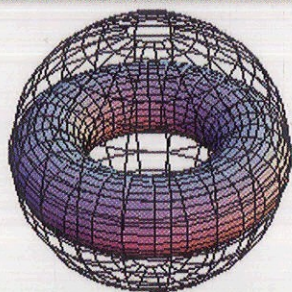


An Integrated Introduction to *Computer Graphics* *and Geometric Modeling*

Ronald Goldman



 **CRC Press**
Taylor & Francis Group

A CHAPMAN & HALL BOOK

Contents

Foreword	xv
Dedication	xvii
Preface	xix
Author	xxix

I Two-Dimensional Computer Graphics: From Common Curves to Intricate Fractals

1 Turtle Graphics	3
1.1 Turtle Graphics	3
1.2 Turtle Commands.....	4
1.3 Turtle Programs.....	7
1.4 Summary	9
Exercises	9
2 Fractals from Recursive Turtle Programs	13
2.1 Fractals	13
2.2 Looping Lemmas	13
2.3 Fractal Curves and Recursive Turtle Programs	17
2.3.1 Fractal Gaskets	17
2.3.2 Bump Fractals	19
2.4 Summary: Fractals—Recursion Made Visible	20
Exercises	21
Programming Projects	23
3 Some Strange Properties of Fractal Curves	29
3.1 Fractal Strangeness	29
3.2 Dimension	29
3.2.1 Fractal Dimension	31
3.2.2 Computing Fractal Dimension from Recursive Turtle Programs	32
3.3 Differentiability	32
3.4 Attraction.....	34
3.4.1 Base Cases for the Sierpinski Gasket	34
3.4.2 Base Cases for the Koch Curve	35
3.4.3 Attractors	36
3.5 Summary	36
Exercises	37

4	Affine Transformations	39
4.1	Transformations	39
4.2	Conformal Transformations	40
4.2.1	Translation	40
4.2.2	Rotation	41
4.2.3	Uniform Scaling	42
4.3	The Algebra of Affine Transformations	43
4.4	The Geometry of Affine Transformations	44
4.5	Affine Coordinates and Affine Matrices	45
4.6	Conformal Transformations: Revisited	46
4.7	General Affine Transformations	46
4.7.1	Image of One Point and Two Linearly Independent Vectors	47
4.7.2	Nonuniform Scaling	48
4.7.3	Image of Three Noncollinear Points	50
4.8	Summary	51
4.8.1	Affine Transformations and Affine Coordinates	51
4.8.2	Matrices for Affine Transformations in the Plane	53
	Exercises	54
5	Affine Geometry: A Connect-the-Dots Approach to Two-Dimensional Computer Graphics	61
5.1	Two Shortcomings of Turtle Graphics	61
5.2	Affine Graphics	62
5.2.1	The CODO Language	62
5.2.2	Sample CODO Programs	63
5.3	Summary	65
	Exercises	67
6	Fractals from Iterated Function Systems	71
6.1	Generating Fractals by Iterating Transformations	71
6.2	Fractals as Fixed Points of Iterated Function Systems	73
6.3	Fractals as Attractors	74
6.4	Fractals with Condensation Sets	74
6.5	Summary	76
	Exercises	77
	Programming Project	79
7	The Fixed-Point Theorem and Its Consequences	81
7.1	Fixed Points and Iteration	81
7.2	The Trivial Fixed Point Theorem	82
7.3	Consequences of the Trivial Fixed-Point Theorem	84
7.3.1	Root Finding Methods	84
7.3.2	Relaxation Methods	87
7.3.3	Fractals	89
7.3.3.1	Compact Sets and the Hausdorff Metric	90
7.3.3.2	Contractive Transformations and Iterated Function Systems	91
7.3.3.3	Fractal Theorem, Fractal Algorithm, and Fractal Strategy	92
7.4	Summary	93
	Exercises	94
	Programming Projects	98

8	Recursive Turtle Programs and Conformal Iterated Function Systems	101
8.1	Motivating Questions	101
8.2	The Effect of Changing the Turtle's Initial State	101
8.3	Equivalence Theorems	103
8.4	Conversion Algorithms	106
	8.4.1 Ron's Algorithm	106
	8.4.2 Tao's Algorithm	107
8.5	Bump Fractals	109
8.6	Summary	110
	Exercises	111
	Programming Projects	113

II Mathematical Methods for Three-Dimensional Computer Graphics

9	Vector Geometry: A Coordinate-Free Approach	117
9.1	Coordinate-Free Methods	117
9.2	Vectors and Vector Spaces	118
9.3	Points and Affine Spaces	119
9.4	Vector Products	120
	9.4.1 Dot Product	121
	9.4.2 Cross Product	122
	9.4.3 Determinant	123
9.5	Summary	123
	Appendix A: The Nonassociativity of the Cross Product	124
	Appendix B: The Algebra of Points and Vectors	126
	Exercises	128
10	Coordinate Algebra	131
10.1	Rectangular Coordinates	131
10.2	Addition, Subtraction, and Scalar Multiplication	131
10.3	Vector Products	132
	10.3.1 Dot Product	133
	10.3.2 Cross Product	133
	10.3.3 Determinant	134
10.4	Summary	134
	Exercises	135
11	Some Applications of Vector Geometry	139
11.1	Introduction	139
11.2	Trigonometric Laws	139
	11.2.1 Law of Cosines	139
	11.2.2 Law of Sines	140
11.3	Representations for Lines and Planes	141
	11.3.1 Lines	141
	11.3.2 Planes	141

11.4	Metric Formulas.....	142
11.4.1	Distance.....	142
11.4.1.1	Distance between Two Points.....	142
11.4.1.2	Distance between a Point and a Line.....	143
11.4.1.3	Distance between a Point and a Plane.....	143
11.4.1.4	Distance between Two Lines.....	144
11.4.2	Area.....	145
11.4.2.1	Triangles and Parallelograms.....	145
11.4.2.2	Polygons: Newell's Formula.....	146
11.4.3	Volume.....	147
11.5	Intersection Formulas for Lines and Planes.....	148
11.5.1	Intersecting Two Lines.....	148
11.5.2	Intersecting Three Planes.....	149
11.5.3	Intersecting Two Planes.....	150
11.6	Spherical Linear Interpolation.....	151
11.7	Inside-Outside Tests.....	153
11.7.1	Ray Casting.....	153
11.7.2	Winding Number.....	154
11.8	Summary.....	156
11.8.1	Trigonometric Laws.....	156
11.8.2	Metric Formulas.....	157
11.8.2.1	Distance.....	157
11.8.2.2	Area.....	157
11.8.2.3	Volume.....	158
11.8.3	Intersections.....	158
11.8.4	Interpolation.....	158
11.8.5	Winding Number.....	158
	Exercises.....	159

12 Coordinate-Free Formulas for Affine and Projective Transformations.....

	Transformations.....	163
12.1	Transformations for Three-Dimensional Computer Graphics.....	163
12.2	Affine and Projective Transformations.....	163
12.3	Rigid Motions.....	164
12.3.1	Translation.....	165
12.3.2	Rotation.....	165
12.3.3	Mirror Image.....	167
12.4	Scaling.....	168
12.4.1	Uniform Scaling.....	169
12.4.2	Nonuniform Scaling.....	169
12.5	Projections.....	170
12.5.1	Orthogonal Projection.....	171
12.5.2	Perspective.....	171
12.6	Summary.....	173
12.6.1	Affine and Projective Transformations without Matrices.....	173
12.6.2	Formulas for Affine and Projective Transformations.....	174
	Exercises.....	174

13	Matrix Representations for Affine and Projective Transformations	179
13.1	Matrix Representations for Affine Transformations	179
13.2	Linear Transformation Matrices and Translation Vectors	181
13.2.1	Linear Transformation Matrices	182
13.2.2	Translation Vectors	183
13.3	Rigid Motions	183
13.3.1	Translation	183
13.3.2	Rotation	184
13.3.3	Mirror Image	185
13.4	Scaling	187
13.4.1	Uniform Scaling	187
13.4.2	Nonuniform Scaling	187
13.5	Projections	188
13.5.1	Orthogonal Projection	189
13.6	Perspective	189
13.6.1	Projective Transformations and Homogeneous Coordinates	190
13.6.2	Matrices for Perspective Projections	191
13.7	Summary	193
13.7.1	Matrix Representations for Affine and Projective Transformations	193
13.7.2	Matrices for Affine and Projective Transformations	194
	Exercises	195
	Programming Projects	199
14	Projective Space versus the Universal Space of Mass-Points	205
14.1	Algebra and Geometry	205
14.2	<i>Projective Space: The Standard Model</i>	206
14.3	Mass-Points: The Universal Model	210
14.4	Perspective and Pseudoperspective	213
14.4.1	Perspective and the Law of the Lever	213
14.4.2	Pseudoperspective and Pseudodepth	214
14.5	Summary	218
	Exercises	219
15	Quaternions: Multiplication in the Space of Mass-Points	223
15.1	Vector Spaces and Division Algebras	223
15.2	Complex Numbers	224
15.3	Quaternions	227
15.3.1	Quaternion Multiplication	227
15.3.2	Quaternion Representations for Conformal Transformations	230
15.3.3	Quaternions versus Matrices	232
15.3.4	Avoiding Distortion	233
15.3.5	Key Frame Animation	234
15.3.6	Conversion Formulas	235
15.4	Summary	238
	Exercises	239
	Programming Projects	245

III Three-Dimensional Computer Graphics: Realistic Rendering

16	Color and Intensity	249
16.1	Motivation	249
16.2	The RGB Color Model	249
16.3	Ambient Light	250
16.4	Diffuse Reflection	251
16.5	Specular Reflection	252
16.6	Total Intensity	254
16.7	Summary	255
	Exercises	256
17	Recursive Ray Tracing	257
17.1	Raster Graphics	257
17.2	Recursive Ray Tracing	257
17.3	Shadows	259
17.4	Reflection	260
17.5	Refraction	261
17.6	Summary	264
	Exercises	265
18	Surfaces I: The General Theory	267
18.1	Surface Representations	267
18.1.1	Implicit Surfaces	267
18.1.2	Parametric Surfaces	267
18.1.3	Deformed Surfaces	268
18.1.4	Procedural Surfaces	268
18.2	Surface Normals	269
18.2.1	Implicit Surfaces	269
18.2.2	Parametric Surfaces	269
18.2.3	Deformed Surfaces	270
18.3	Ray-Surface Intersections	272
18.3.1	Implicit Surfaces	272
18.3.2	Parametric Surfaces	272
18.3.3	Deformed Surfaces	273
18.4	Mean and Gaussian Curvature	274
18.4.1	Implicit Surfaces	274
18.4.2	Parametric Surfaces	275
18.4.3	Deformed Surfaces	275
18.5	Summary	275
18.5.1	Implicit Surfaces	276
18.5.2	Parametric Surfaces	276
18.5.3	Deformed Surfaces	278
	Exercises	278
19	Surfaces II: Simple Surfaces	281
19.1	Simple Surfaces	281
19.2	Intersection Strategies	281
19.3	Planes and Polygons	282

19.4	Natural Quadrics	284
19.4.1	Spheres	284
19.4.1.1	Intersecting a Line and a Circle	285
19.4.1.2	Inversion Formulas for the Line	285
19.4.2	Cylinders	287
19.4.2.1	Intersecting a Line and an Infinite Cylinder	287
19.4.2.2	Intersecting a Line and a Bounded Cylinder	289
19.4.3	Cones	290
19.4.4	Ellipsoids, Elliptical Cylinders, and Elliptical Cones	292
19.5	General Quadric Surfaces	292
19.6	Tori	295
19.6.1	Bounding the Torus	298
19.7	Surfaces of Revolution	299
19.8	Summary	303
	Exercises	304
	Programming Projects	306
20	Solid Modeling	309
20.1	Solids	309
20.2	Constructive Solid Geometry (CSG)	309
20.3	Boundary Representations (B-Rep)	313
20.4	Octrees	317
20.5	Summary	319
	Exercises	319
	Programming Projects	322
21	Shading	325
21.1	Polygonal Models	325
21.1.1	Newell's Formula for the Normal to a Polygon	326
21.2	Uniform Shading	326
21.3	Gouraud Shading	327
21.4	Phong Shading	331
21.4.1	Naive Phong Shading	331
21.4.2	Fast Phong Shading and Diffuse Reflection	332
21.4.3	Fast Phong Shading and Specular Reflection	334
21.4.4	Phong Shading and Spherical Linear Interpolation	335
21.5	Summary	337
	Exercises	339
	Programming Project	339
22	Hidden Surface Algorithms	341
22.1	Hidden Surface Algorithms	341
22.2	The Heedless Painter	342
22.3	z-Buffer (Depth Buffer)	342
22.4	Scan Line	343
22.5	Ray Casting	346
22.6	Depth Sort	347
22.6.1	Polygon Splitting	350
22.7	BSP-Tree	351

22.8	Summary	352
	Exercises	352
	Programming Projects.....	353
23	Radiosity	355
23.1	Radiosity	355
23.2	The Radiosity Equations.....	355
23.2.1	The Rendering Equation	356
23.2.2	The Radiosity Equation: <i>Continuous Form</i>	356
23.2.3	The Radiosity Equation: <i>Discrete Form</i>	359
23.3	Form Factors	361
23.3.1	Hemi-Cubes.....	363
23.4	The Radiosity Rendering Algorithm	366
23.5	Solving the Radiosity Equations.....	368
23.5.1	Gathering.....	368
23.5.2	Shooting: Progressive Refinement	370
23.6	Summary	372
	Exercises	373
	Programming Project	375

IV Geometric Modeling: Freedom Curves and Surfaces

24	Bezier Curves and Surfaces	379
24.1	Interpolation and Approximation.....	379
24.2	The de Casteljau Evaluation Algorithm	380
24.3	The Bernstein Representation	383
24.4	Geometric Properties of Bezier Curves	384
24.4.1	Affine Invariance.....	385
24.4.2	Convex Hull Property	386
24.4.3	Variation Diminishing Property	386
24.4.4	Interpolation of the First and Last Control Points.....	387
24.5	Differentiating the de Casteljau Algorithm.....	388
24.5.1	Smoothly Joining Two Bezier Curves.....	389
24.5.2	Uniqueness of the Bezier Control Points.....	390
24.6	Tensor Product Bezier Patches	391
24.7	Summary	395
	Exercises	397
25	Bezier Subdivision.....	401
25.1	Divide and Conquer.....	401
25.2	The de Casteljau Subdivision Algorithm.....	401
25.3	Rendering and Intersection Algorithms.....	405
25.3.1	Rendering and Intersecting Bezier Curves	405
25.3.2	Rendering and Intersecting Bezier Surfaces	407
25.4	The Variation Diminishing Property of Bezier Curves.....	409
25.5	Joining Bezier Curves Smoothly	410
25.6	Summary	411
	Exercises	412
	Programming Projects.....	414

26	Blossoming	417
	26.1 Motivation	417
	26.2 The Blossom	418
	26.3 Blossoming and the de Casteljau Algorithm	419
	26.3.1 Bezier Subdivision from Blossoming	422
	26.4 Differentiation and the Homogeneous Blossom	423
	26.4.1 Homogenization and the Homogeneous Blossom	423
	26.4.2 Differentiating the de Casteljau Algorithm	427
	26.4.3 Conversion Algorithms between Monomial and Bezier Form	430
	26.5 Summary	431
	Exercises	434
27	B-Spline Curves and Surfaces	437
	27.1 Motivation	437
	27.2 Blossoming and the Local de Boor Algorithm	438
	27.3 B-Spline Curves and the Global de Boor Algorithm	441
	27.4 Smoothness	443
	27.5 Labeling and Locality in the Global de Boor Algorithm	445
	27.6 Every Spline is a B-Spline	446
	27.7 Geometric Properties of B-Spline Curves	448
	27.8 Tensor Product B-Spline Surfaces	449
	27.9 Non-Uniform Rational B-Splines (NURBS)	451
	27.10 Summary	452
	Exercises	453
28	Knot Insertion Algorithms for B-Spline Curves and Surfaces	457
	28.1 Motivation	457
	28.2 Knot Insertion	457
	28.3 Local Knot Insertion Algorithms	458
	28.3.1 Boehm's Knot Insertion Algorithm	458
	28.3.2 The Oslo Algorithm	460
	28.3.3 Conversion from B-Spline to Piecewise Bezier Form	461
	28.3.4 The Variation Diminishing Property for B-Spline Curves	461
	28.3.5 Algorithms for Rendering and Intersecting B-Spline Curves and Surfaces	463
	28.4 Global Knot Insertion Algorithms	464
	28.4.1 The Lane–Riesenfeld Algorithm	464
	28.4.2 Schaefer's Knot Insertion Algorithm	467
	28.4.3 Convergence of Knot Insertion Algorithms	468
	28.4.4 Algorithms for Rendering and Intersecting B-Spline Curves and Surfaces Revisited	470
	28.5 Summary	471
	Exercises	474
	Programming Project	475
29	Subdivision Matrices and Iterated Function Systems	477
	29.1 <i>Subdivision Algorithms and Fractal Procedures</i>	477
	29.2 Subdivision Matrices	478
	29.2.1 Subdivision Matrices for Bezier Curves	479
	29.2.2 Subdivision Matrices for Uniform B-Spline Curves	481

29.3	Iterated Function Systems Built from Subdivision Matrices.....	485
29.3.1	Lifting the Control Points to Higher Dimensions.....	485
29.3.2	Normal Curves.....	489
29.4	Fractals with Control Points.....	491
29.5	Summary.....	493
29.5.1	Bezier Curves.....	494
29.5.2	Uniform B-Splines.....	495
	Exercises.....	496
	Programming Projects.....	497
30	Subdivision Surfaces.....	499
30.1	Motivation.....	499
30.2	Box Splines.....	500
30.2.1	Split and Average.....	500
30.2.2	A Subdivision Procedure for Box Spline Surfaces.....	500
30.3	Quadrilateral Meshes.....	503
30.3.1	Centroid Averaging.....	505
30.3.1.1	Uniform Bicubic B-Spline Surfaces.....	505
30.3.1.2	Arbitrary Quadrilateral Meshes.....	507
30.3.2	Stencils.....	509
30.3.2.1	Stencils for Uniform B-Splines.....	509
30.3.2.2	Stencils for Extraordinary Vertices.....	511
30.4	Triangular Meshes.....	512
30.4.1	Centroid Averaging for Triangular Meshes.....	512
30.4.1.1	Three-Direction Quartic Box Splines.....	512
30.4.1.2	Arbitrary Triangular Meshes.....	514
30.4.2	Stencils for Triangular Meshes.....	516
30.4.2.1	Stencils for Three-Direction Quartic Box Splines.....	516
30.4.2.2	Stencils for Extraordinary Vertices.....	517
30.5	Summary.....	518
30.5.1	Bicubic Tensor Product B-Splines and Three-Direction Quartic Box Splines.....	518
30.5.1.1	Split and Average.....	519
30.5.1.2	Centroid Averaging.....	519
30.5.1.3	Stencils.....	521
30.5.2	Centroid Averaging for Meshes of Arbitrary Topology.....	521
30.5.3	Stencils for Extraordinary Vertices.....	523
	Exercises.....	524
	Programming Projects.....	527
	Further Readings.....	529
	Index.....	533