

An abstract geometric pattern consisting of numerous red circles of varying sizes, arranged in vertical columns and connected by thin red lines. The pattern is set against a yellow background and appears to be a stylized representation of a quantum system or a lattice structure. The circles are arranged in a way that suggests a complex, interconnected network, with some circles being larger than others, possibly representing different energy levels or states. The lines connect the circles in a way that forms a series of overlapping shapes, creating a sense of depth and movement.

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DYNAMICS OF
One-Dimensional
Quantum Systems

Inverse-Square Interaction Models

CAMBRIDGE

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