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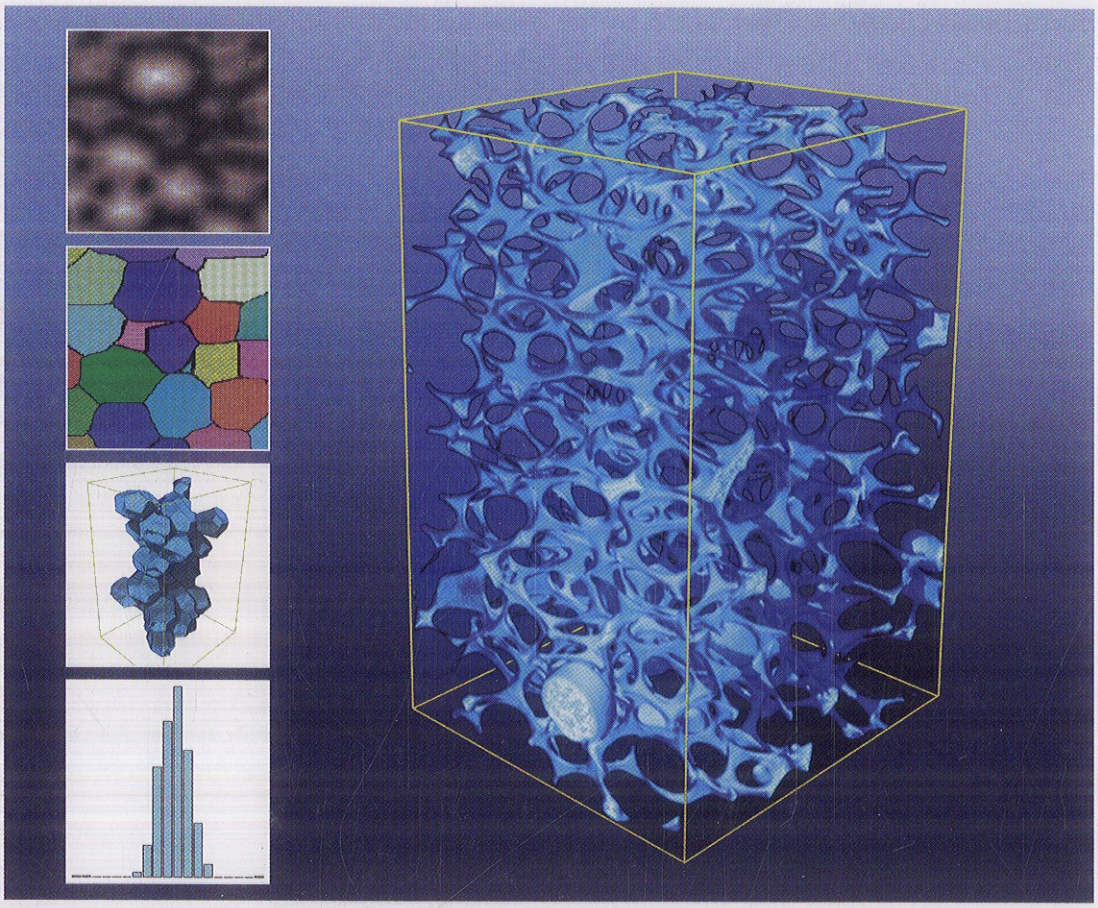
WILEY-VCH

3D Images of Material Structures

Processing and Analysis



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