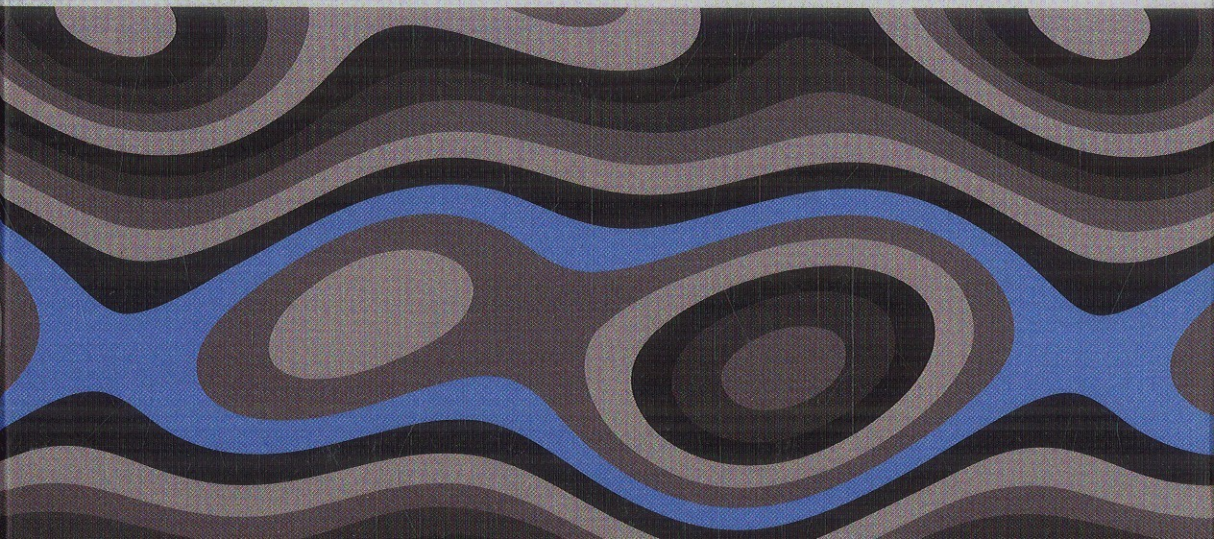


BIRKHAUSER

Advances in
Mathematical
Fluid Mechanics

Eduard Feireisl
Antonín Novotný

Singular Limits in Thermodynamics of Viscous Fluids



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