

# The Scalar–Tensor Theory of Gravitation

YASUNORI FUJII  
KEI-ICHI MAEDA

CAMBRIDGE MONOGRAPHS  
ON MATHEMATICAL PHYSICS

# Contents

<i>Preface</i>	<i>xi</i>
<i>Conventions and notation</i>	<i>xiv</i>
<b>1 Introduction</b>	<b>1</b>
1.1 What is the scalar-tensor theory of gravitation?	2
1.2 Where does the scalar field come from?	9
1.2.1 The scalar field arising from the size of compactified internal space	9
1.2.2 The dilaton from string theory	12
1.2.3 The scalar field in a brane world	16
1.2.4 The scalar field in the assumed two-sheeted structure of space-time	22
1.3 Comments	25
1.3.1 The weak equivalence principle	25
1.3.2 The value of $\omega$ and mass of the scalar field	28
1.3.3 Conformal transformation	29
1.3.4 Mach's principle and Dirac's suggestion for time-dependent $G$	31
1.3.5 Does the scalar-tensor theory have any advantage over simple scalar theories?	34
<b>2 The prototype Brans-Dicke model</b>	<b>37</b>
2.1 The Lagrangian	38
2.2 Field equations	40
2.3 The weak-field approximation	43
2.4 The parameterized post-Newtonian approximation	47

2.5	A spinor field as matter	53
2.6	The mechanism of mixing	57
<b>3</b>	<b>Conformal transformation</b>	<b>61</b>
3.1	What is a conformal transformation?	62
3.2	Nonminimal coupling	66
3.3	Coupling to matter	71
3.4	The geodesic in the E frame	74
<b>4</b>	<b>Cosmology with <math>\Lambda</math></b>	<b>77</b>
4.1	How has the cosmological constant re-emerged?	77
4.2	The standard theory with $\Lambda$	82
4.3	The prototype BD model without $\Lambda$	84
4.4	The prototype BD model with $\Lambda$	88
4.4.1	Solution in the J frame	88
4.4.2	Solution in the E frame	91
4.4.3	A proposed revision and remarks	101
<b>5</b>	<b>Models of an accelerating universe</b>	<b>105</b>
5.1	Dark energy	106
5.2	Quintessence	110
5.3	Quintessence in the brane world	115
5.3.1	A scalar field in the brane world	116
5.3.2	Quintessence in the brane world	118
5.4	Scalar-tensor theory	124
5.4.1	Hesitation behavior	124
5.4.2	A two-scalar model	128
5.4.3	Qualitative features unique to the two-scalar model	139
<b>6</b>	<b>Quantum effects</b>	<b>145</b>
6.1	Scale invariance	147
6.2	The dilaton as a Nambu–Goldstone boson	151
6.3	The contribution from loops	155
6.3.1	The coupling to matter of $\sigma$ , a self-coupled scalar field	155
6.3.2	The coupling to matter of $\sigma$ , scalar QED	160
6.4	Non-Newtonian gravity	168
6.5	A consequence of the suspected occurrence of the scalar field in the Maxwell term	177
6.6	Time-variability of the fine-structure constant	179
6.6.1	Electromagnetic coupling of $\Phi_*$	180
6.6.2	The behavior of the scalar field	184

Appendix A: The scalar field from Kaluza–Klein theory	187
Appendix B: The curvature scalar from the assumed two-sheeted space-time	192
Appendix C: The field equation of gravity in the presence of nonminimal coupling	195
Appendix D: The law of conservation of matter	198
Appendix E: Eddington's parameters	200
Appendix F: Conformal transformation of a spinor field	204
Appendix G: Conformal transformation of the curvature scalar	209
Appendix H: A special choice for conformal invariance	214
Appendix J: The matter energy–momentum nonconservation law in the E frame	216
Appendix K: A modification to the $\Lambda$ term	217
Appendix L: Einstein's equation in the brane world	219
Appendix M: Dilatation current	223
Appendix N: Loop integrals in continuous dimensions	225
Appendix O: A conformal frame in which particle masses are finally constant	227
<i>References</i>	230
<i>Index</i>	237