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Solid Mechanics
and its Applications

Nonlinear Solid Mechanics

Theoretical Formulations and
Finite Element Solution Methods

 Springer

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