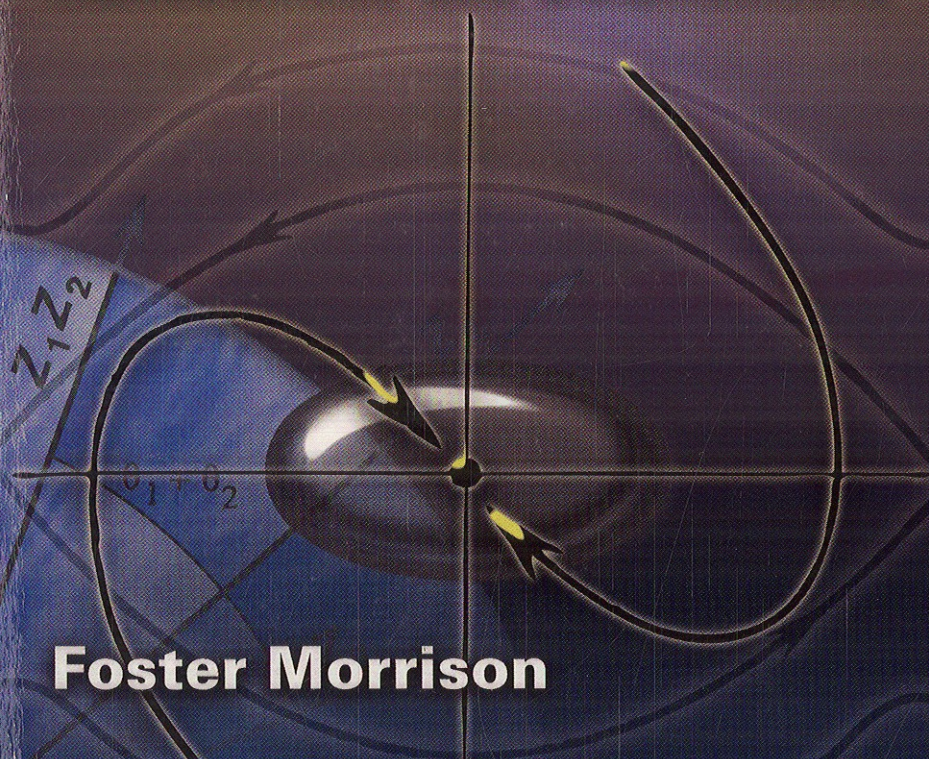


# The Art of Modeling Dynamic Systems

Forecasting for Chaos,  
Randomness and Determinism



**Foster Morrison**

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