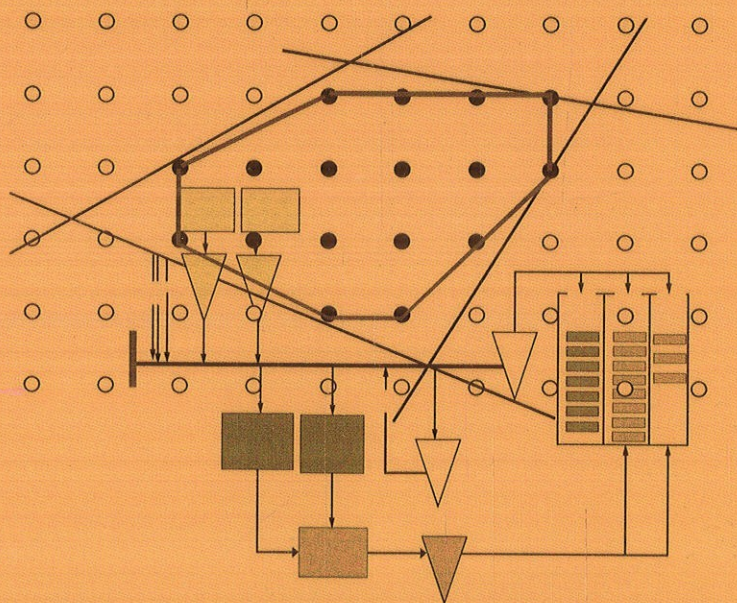


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Production Planning by Mixed Integer Programming



Springer

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