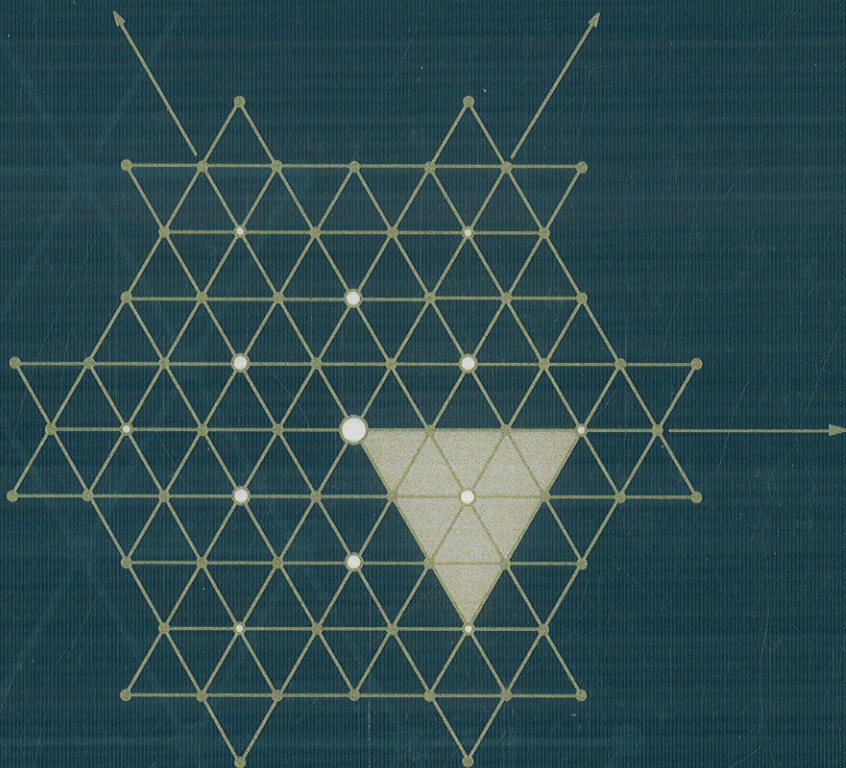


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Introduction to the
MATHEMATICS
of **SUBDIVISION**
SURFACES

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