

Semyon G. Rabinovich

# Evaluating Measurement Accuracy

*A Practical Approach*



Springer

# Contents

|          |                                                                     |     |
|----------|---------------------------------------------------------------------|-----|
| <b>1</b> | <b>General Concepts in the Theory of Measurements</b> .....         | 1   |
| 1.1      | Basic Concepts and Terms .....                                      | 1   |
| 1.2      | The Basic Metrological Problems .....                               | 4   |
| 1.3      | New Forms of International Cooperation in Metrology .....           | 10  |
| 1.4      | Postulates of the Theory of Measurements .....                      | 12  |
| 1.5      | Classification of Measurements .....                                | 18  |
| 1.6      | Classification of Measurements Errors .....                         | 22  |
| 1.7      | General Approach to Evaluation of Measurement Inaccuracy .....      | 25  |
| 1.8      | Presentation of Measurement Results .....                           | 26  |
| <b>2</b> | <b>Measuring Instruments and Their Properties</b> .....             | 31  |
| 2.1      | Types of Measuring Instruments .....                                | 31  |
| 2.2      | Metrological Characteristics of Measuring Instruments .....         | 33  |
| 2.3      | Rating of the Errors of Measuring Instruments .....                 | 36  |
| 2.4      | Dynamic Characteristics of Measuring Instruments .....              | 49  |
| 2.5      | Calibration and Verification of Measuring Instruments .....         | 53  |
| 2.6      | Designing a Calibration Scheme .....                                | 58  |
| 2.7      | Statistical Analysis of Measuring Instrument Errors .....           | 65  |
| <b>3</b> | <b>Statistical Methods for Experimental Data Processing</b> .....   | 71  |
| 3.1      | Methods for Describing Random Quantities .....                      | 71  |
| 3.2      | Requirements for Statistical Estimates .....                        | 75  |
| 3.3      | Evaluation of the Parameters of the Normal Distribution .....       | 76  |
| 3.4      | Elimination of Outlying Data .....                                  | 80  |
| 3.5      | Construction of Confidence Intervals .....                          | 82  |
| 3.6      | Testing Hypotheses about the Form of the Distribution Function .... | 87  |
| 3.7      | Testing for Homogeneity of Samples .....                            | 89  |
| 3.8      | Robust Estimates .....                                              | 95  |
| 3.9      | Application of the Bayes' Theorem .....                             | 99  |
| <b>4</b> | <b>Direct Measurements</b> .....                                    | 101 |
| 4.1      | Relation Between Single and Multiple Measurements .....             | 101 |
| 4.2      | Classification of Elementary Errors .....                           | 104 |

|          |                                                                                                 |            |
|----------|-------------------------------------------------------------------------------------------------|------------|
| 4.3      | Modeling of Elementary Errors .....                                                             | 108        |
| 4.3.1    | Absolutely Constant Errors .....                                                                | 109        |
| 4.3.2    | Conditionally Constant Errors .....                                                             | 109        |
| 4.3.3    | Purely Random Errors .....                                                                      | 110        |
| 4.3.4    | Quasirandom Errors .....                                                                        | 110        |
| 4.4      | Composition of Uniform Distributions .....                                                      | 111        |
| 4.5      | Methods for Precise Measurements .....                                                          | 117        |
| 4.6      | Accuracy of Single Measurements Using Measuring<br>Instruments Under Reference Conditions ..... | 119        |
| 4.7      | Accuracy of Single Measurements Using Measuring<br>Instruments Under Rated Conditions .....     | 122        |
| 4.8      | Accuracy of Multiple Measurements .....                                                         | 128        |
| 4.9      | Comparison of Different Methods for Combining<br>Systematic and Random Errors .....             | 136        |
| <b>5</b> | <b>Indirect Measurements .....</b>                                                              | <b>141</b> |
| 5.1      | Terminology and Classification .....                                                            | 141        |
| 5.2      | Correlation Coefficient and Its Calculation .....                                               | 142        |
| 5.3      | The Traditional Method of Experimental Data Processing .....                                    | 145        |
| 5.4      | Merits and Shortcomings of the Traditional Method .....                                         | 149        |
| 5.5      | The Method of Reduction .....                                                                   | 152        |
| 5.6      | The Method of Transformation .....                                                              | 154        |
| 5.7      | Total Uncertainty of Indirect Measurements .....                                                | 161        |
| 5.8      | Accuracy of Single Indirect Measurements .....                                                  | 166        |
| 5.9      | Accuracy of a Single Measurement with a Chain of Instruments .....                              | 167        |
| 5.10     | Application of the Monte Carlo Method .....                                                     | 169        |
| <b>6</b> | <b>Combined and Simultaneous Measurements .....</b>                                             | <b>173</b> |
| 6.1      | General Remarks About the Method of Least Squares .....                                         | 173        |
| 6.2      | Measurements with Linear Equally Accurate<br>Conditional Equations .....                        | 175        |
| 6.3      | Measurements with Linear Unequally Accurate<br>Conditional Equations .....                      | 178        |
| 6.4      | Linearization of Nonlinear Conditional Equations .....                                          | 180        |
| 6.5      | Examples of the Application of the Method of Least Squares .....                                | 181        |
| 6.6      | General Remarks on Determination of the Parameters<br>in Formulas from Empirical Data .....     | 186        |
| 6.7      | Construction of Transfer Functions of Measuring Transducers .....                               | 188        |
| <b>7</b> | <b>Combining the Results of Measurements .....</b>                                              | <b>193</b> |
| 7.1      | Introductory Remarks .....                                                                      | 193        |
| 7.2      | Theoretical Principles .....                                                                    | 193        |
| 7.3      | Effect of the Error of the Weights on the Error<br>of the Weighted Mean .....                   | 197        |

|                         |                                                                                          |            |
|-------------------------|------------------------------------------------------------------------------------------|------------|
| 7.4                     | Combining the Results of Measurements with Predominately Random Errors .....             | 199        |
| 7.5                     | Combining the Results of Measurements Containing Both Systematic and Random Errors ..... | 201        |
| 7.6                     | Combining the Results of Single Measurements .....                                       | 204        |
| <b>8</b>                | <b>Examples of Measurements and Measurement Data Processing .....</b>                    | <b>209</b> |
| 8.1                     | Voltage Measurement with a Pointer-Type Voltmeter .....                                  | 209        |
| 8.1.1                   | A Priori Estimation of Measurement Inaccuracy .....                                      | 209        |
| 8.1.2                   | Universal Estimation of Measurement Inaccuracy .....                                     | 212        |
| 8.1.3                   | Individual Estimation of Measurement Inaccuracy .....                                    | 214        |
| 8.2                     | Voltage Measurement with a Potentiometer and a Voltage Divider .....                     | 215        |
| 8.3                     | Comparison of Mass Measures .....                                                        | 220        |
| 8.4                     | Measurement of Electric Power at High Frequency .....                                    | 221        |
| 8.5                     | An Indirect Measurement of the Electrical Resistance of a Resistor .....                 | 222        |
| 8.5.1                   | Application of the Traditional Method .....                                              | 222        |
| 8.5.2                   | Application of the Method of Reduction .....                                             | 225        |
| 8.6                     | Measurement of the Density of a Solid Body .....                                         | 226        |
| 8.6.1                   | Application of the Traditional Method .....                                              | 226        |
| 8.6.2                   | Application of the Method of Transformation .....                                        | 229        |
| 8.7                     | Measurement of Ionization Current .....                                                  | 233        |
| 8.8                     | Measurement of the Activity of a Radioactive Source .....                                | 235        |
| <b>9</b>                | <b>Conclusion .....</b>                                                                  | <b>239</b> |
| 9.1                     | Measurement Data Processing: Past, Present, and Next Steps .....                         | 239        |
| 9.2                     | Remarks on the International Vocabulary of Metrology .....                               | 241        |
| 9.3                     | Drawbacks of the “Guide to the Expression of Uncertainty in Measurement” .....           | 244        |
| <b>Appendix</b> .....   |                                                                                          | <b>249</b> |
| <b>Glossary</b> .....   |                                                                                          | <b>255</b> |
| <b>References</b> ..... |                                                                                          | <b>261</b> |
| <b>Index</b> .....      |                                                                                          | <b>265</b> |