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SPRINGER SERIES IN OPERATIONS  
RESEARCH AND FINANCIAL ENGINEERING

# Principles of Inventory Management

When You Are Down to Four, Order More

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