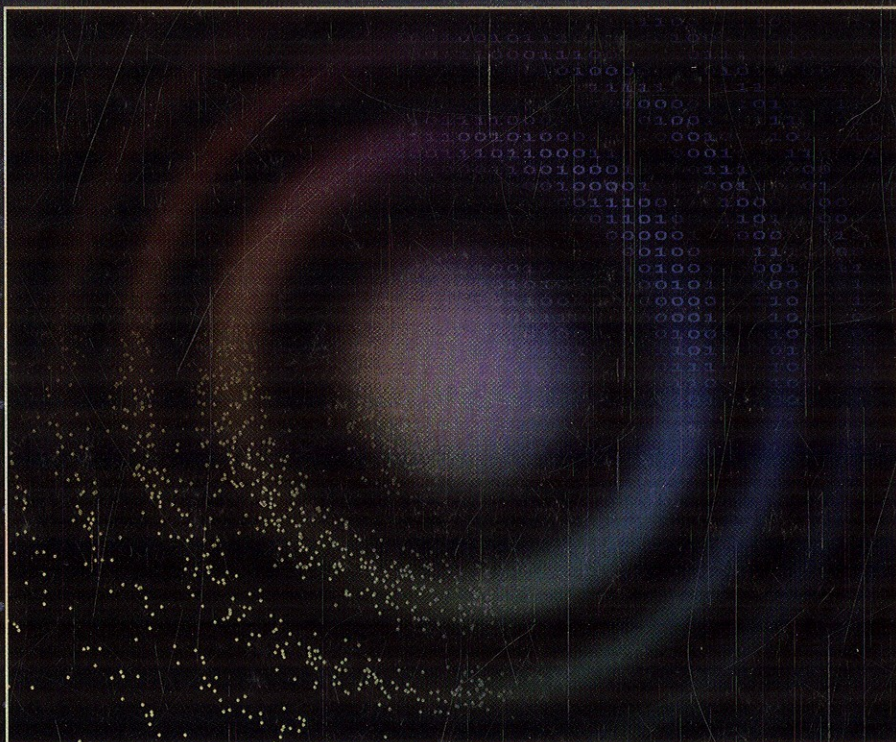


# QUANTUM PROCESSES SYSTEMS, & INFORMATION



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*Preface*

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