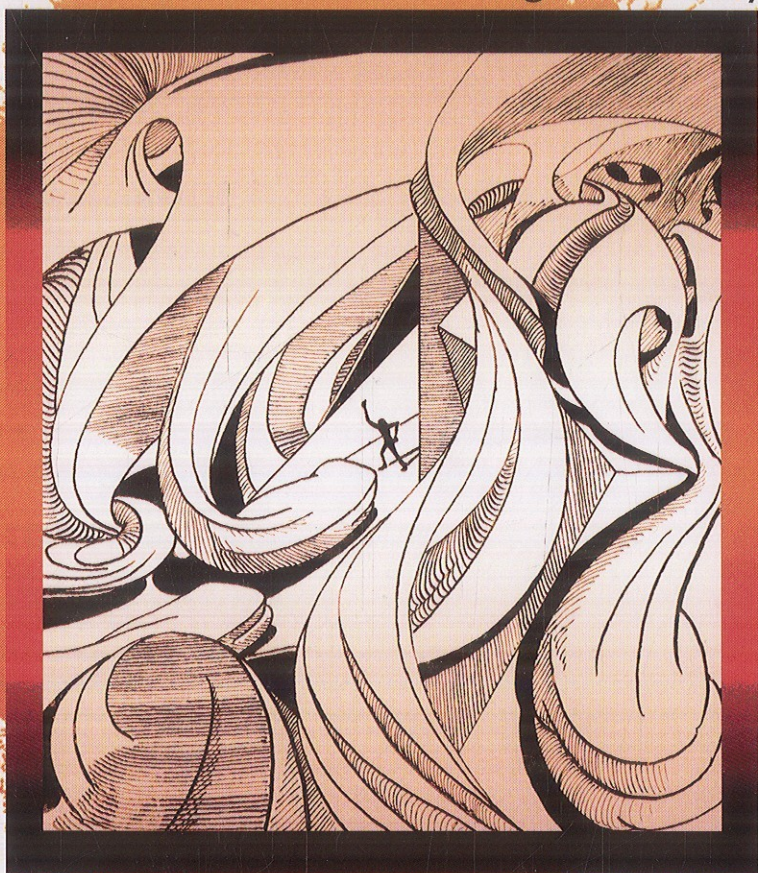


Understanding Game Theory

Introduction to the Analysis of
Many Agent Systems with Competition and Cooperation

Vassili N Kolokoltsov • Oleg A Malafeyev



Contents

<i>Preface</i>	vii
----------------	-----

Basic ideas	1
--------------------	----------

1. Around the prisoner's dilemma	3
1.1 What is a two-player game?	3
1.2 Prisoner's dilemma. Dominated strategies and Pareto optimality	7
1.3 Prisoner's dilemma for crooks, warriors and opera lovers .	9
1.4 Discrete duopoly models and common pool resources; public goods	12
1.5 Common knowledge, rationality and iterated elimination of strictly dominated strategies	13
1.6 Weak dominance; debtors and creditors	15
1.7 Nash equilibrium	16
1.8 Battle of the sexes and Kant's categorical imperative . . .	19
1.9 Chicken game and the Cuban missile crisis	21
1.10 Social dilemmas	22
1.11 Guaranteed payoff, minimax strategy, hedge	25
1.12 Utility function	26
1.13 General objectives of game theory; Pascal's wager	27
2. Auctions and networks	29
2.1 Several players; the volunteers' dilemma	29
2.2 An example on iterated elimination of dominated strategies	31
2.3 Second price and increasing bid auctions	32

2.4	Escalating conflicts	34
2.5	Braess paradox	35
2.6	Wardrop equilibria and selfish routing	37
3.	Wise men and businessmen	39
3.1	Wise men and their wives; imp in the bottle	39
3.2	King Solomon's wisdom	41
3.3	Chain store paradox; centipede game	43
3.4	Normal and extensive forms of a game; battle of the sexes revisited	46
3.5	Dynamic games and subgame perfection; pursuit games	48
3.6	Fair division and the ultimatum game	50
3.7	Cooperation by threat and punishment; infinitely repeated games	51
3.8	Computer tournaments; the triumph of the strategy Tit- for-Tat	58
3.9	Logical games; limits of the sequences	59
3.10	Russian Roulette; games with incomplete information	62
4.	Hawk and doves, lions and lambs	65
4.1	Fitness and stability in population biology (general ideas)	65
4.2	Hawk and Dove games as social dilemmas	67
4.3	Mixed strategies, probability and chance	69
4.4	The theorems of Nash and von Neumann	73
4.5	Expectation and risk; St. Petersburg game	74
4.6	Symmetric mixed strategies Nash equilibria	75
4.7	Invasion of mutants and evolutionary stable strategies	77
4.8	The sex ratio game	80
5.	Coalitions and distribution	81
5.1	Distribution of costs and gains; the core of the game	81
5.2	General principles of fair distribution	85
5.3	Utilitarianism and egalitarianism; compromise set	88
5.4	Equilibrium prices	91
5.5	Linear models and linear programming	93
6.	Presidents and dictators	95
6.1	Collective choice; problems of voting	95

6.2	Four examples of voting rules	97
6.3	Criteria of quality of voting rules	99
6.4	The minority principle; dictators	102
7.	At the doors of quantum games	105
7.1	Quantum bits and Schrödinger's cat	105
7.2	Lattices and quantum logic	108
7.3	Rendezvous of Bob and Alice	111
8.	It's party time!	113
8.1	Combinatorial games	113
8.2	Addition and subtraction of games, order structure	117
8.3	Impartial games and Nim numbers	119
8.4	Games as numbers and numbers as games	123

Armed with mathematics 125

9.	A rapid course in mathematical game theory	127
9.1	Three classical examples of Nash equilibria in economics	127
9.2	Mixed strategies for finite games	131
9.3	Evolutionary stable strategies	139
9.4	Replicator dynamics, Nash's fields and stability	141
9.5	<i>Iterative method of solving matrix games</i>	156
9.6	Zero-sum games and linear programming	159
9.7	Backward induction and dynamic programming	161
9.8	Cooperative games: Nucleus and the Shapley vector	167
9.9	Revision exercises	168
9.10	Solutions to revision exercises	173
10.	Examples of game models	179
10.1	A static model of strategic investment	179
10.2	Variations on Cournot's theme: Territorial price building	182
10.3	Models of inspection	184
10.4	A dynamic model of strategic investments	191
10.5	Game theoretic approach to the analysis of colored (or rainbow) options	197

11. Elements of more advanced analysis	211
11.1 Short overview	211
11.2 Two proofs of the Nash-Glikhsberg theorem on the existence of equilibria	212
11.3 Introduction to structural stability	216
11.4 Introduction to abstract differential games	229
11.5 Cooperative games versus zero-sum games	237
11.6 Turnpikes for stochastic games	240
11.7 Games and tropical (or idempotent) mathematics	245
11.8 The first order partial differential equations in multi- criteria optimization problems	253
11.9 General flows of deterministic and stochastic replicator dynamics	259
<i>Bibliography</i>	273
<i>Index</i>	283