

Solutions Manual to Accompany
AN INTRODUCTION TO
NUMERICAL METHODS
AND ANALYSIS

Revised Edition

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