Igor A. Karnovsky
Olga Lebed

Advanced Methods of Structural Analysis

Springer

Contents

Introduction xxi

Part I Statically Determinate Structures

1 Kinematical Analysis of Structures 3
1.1 Classification of Structures by Kinematical Viewpoint 3
1.2 Generation of Geometrically Unchangeable Structures 5
1.3 Analytical Criteria of the Instantaneously Changeable Structures 7
1.4 Degrees of Freedom 11
Problems 13

2 General Theory of Influence Lines 15
2.1 Analytical Method for Construction of Influence Lines 15
2.1.1 Influence Lines for Reactions 16
2.1.2 Influence Lines for Internal Forces 20
2.2 Application of Influence Lines for Fixed and Moving Loads 27
2.2.1 Fixed Loads 27
2.2.2 Moving Loads 30
2.3 Indirect Load Application 33
2.4 Combining of Fixed and Moving Load Approaches 35
2.5 Properties of Influence Lines 36
Problems 37

3 Multispan Beams and Trusses 39
3.1 Multispan Statically Determinate Beams 39
3.1.1 Generation of Multispan Statically Determinate Hinged Beams 39
3.1.2 Interaction Schemes and Load Path 40

3.1.3 Influence Lines for Multispan Hinged Beams 42
3.1.4 Summary 45
3.2 The Generation of Statically Determinate Trusses 47
3.2.1 Simple Trusses 47
3.2.2 Compound Trusses 48
3.2.3 Complex Trusses 49
3.3 Simple Trusses 49
3.4 Trusses with Subdivided Panels 54
3.4.1 Main and Auxiliary Trusses and Load Path 55
3.4.2 Baltimore and Subdivided Warren Trusses 57
3.5 Special Types of Trusses 61
3.5.1 Three-Hinged Trusses 61
3.5.2 Trusses with a Hinged Chain 64
3.5.3 Complex Trusses 68
3.5.4 Summary 70
Problems 72

4 Three-Hinged Arches 77
4.1 Preliminary Remarks 77
4.1.1 Design Diagram of Three-Hinged Arch 77
4.1.2 *Peculiarities of the Arches* 78
4.1.3 Geometric Parameters of Circular and Parabolic Arches 79
4.2 Internal Forces 80
4.3 Influence Lines for Reactions and Internal Forces 86
4.3.1 Influence Lines for Reactions 88
4.3.2 Influence Lines for Internal Forces 88
4.3.3 Application of Influence Lines 92
4.4 Nil Point Method for Construction of Influence Lines 94
4.4.1 Bending Moment 94
4.4.2 Shear Force 95
4.4.3 *Axial Force* 96
4.5 Special Types of Arches 97
4.5.1 Askew Arch 97
4.5.2 Parabolic Arch with Complex Tie 100
Problems 103

5 Cables 109
5.1 Preliminary Remarks 109
5.1.1 Direct and Inverse Problems 110
5.1.2 Fundamental Relationships 111
5.2 Cable with Neglected Self-Weight 113
5.2.1 Cables Subjected to Concentrated Load 113

5.2.2 Cable Subjected to Uniformly Distributed Load ... 116
5.3 Effect of Arbitrary Load on the Thrust and Sag ... 122
5.4 Cable with Self-Weight ... 125
5.4.1 Fundamental Relationships ... 125
5.4.2 Cable with Supports Located at the Same Level ... 127
5.4.3 Cable with Supports Located on the Different Elevations ... 130
5.5 Comparison of Parabolic and Catenary Cables ... 135
5.6 Effect of Axial Stiffness ... 137
5.6.1 Elastic Cable with Concentrated Load ... 137
5.6.2 Elastic Cable with Uniformly Distributed Load ... 139
Problems ... 140

6 Deflections of Elastic Structures ... 145
6.1 Introduction ... 145
6.2 Initial Parameters Method ... 147
6.3 Maxwell–Mohr Method ... 159
6.3.1 Deflections Due to Fixed Loads ... 159
6.3.2 Deflections Due to Change of Temperature ... 165
6.3.3 Summary ... 170
6.4 Displacement Due to Settlement of Supports and Errors of Fabrication ... 170
6.5 Graph Multiplication Method ... 176
6.6 Elastic Loads Method ... 185
6.7 Reciprocal Theorems ... 189
6.7.1 Theorem of Reciprocal Works (Betti Theorem) ... 189
6.7.2 Theorem of Reciprocal Unit Displacements (Maxwell Theorem) ... 190
6.7.3 Theorem of Reciprocal Unit Reactions (Rayleigh First Theorem) ... 192
6.7.4 Theorem of Reciprocal Unit Displacements and Reactions (Rayleigh Second Theorem) ... 193
6.7.5 Summary ... 193
Problems ... 195

Part II Statically Indeterminate Structures

7 The Force Method ... 211
7.1 Fundamental Idea of the Force Method ... 211
7.1.1 Degree of Redundancy, Primary Unknowns and Primary System ... 211
7.1.2 Compatibility Equation in Simplest Case ... 214

7.2 Canonical Equations of Force Method 217
7.2.1 The Concept of Unit Displacements 217
7.2.2 Calculation of Coefficients and Free Terms of Canonical Equations 219
7.3 Analysis of Statically Indeterminate Structures 222
7.3.1 Continuous Beams 222
7.3.2 Analysis of Statically Indeterminate Frames 224
7.3.3 Analysis of Statically Indeterminate Trusses 233
7.3.4 Analysis of Statically Indeterminate Arches 237
7.4 Computation of Deflections of Redundant Structures 243
7.5 Settlements of Supports 246
7.6 Temperature Changes 251
Problems 259

8 The Displacement Method 271
8.1 Fundamental Idea of the Displacement Method 271
8.1.1 Kinematical Indeterminacy 272
8.1.2 Primary System and Primary Unknowns 274
8.1.3 Compatibility Equation. Concept of Unit Reaction 275
8.2 Canonical Equations of Displacement Method 276
8.2.1 Compatibility Equations in General Case 276
8.2.2 Calculation of Unit Reactions 277
8.2.3 Properties of Unit Reactions 279
8.2.4 Procedure for Analysis 280
8.3 Comparison of the Force and Displacement Methods 291
8.3.1 Properties of Canonical Equations 292
8.4 Sidesway Frames with Absolutely Rigid Crossbars 294
8.5 Special Types of Exposures 296
8.5.1 Settlements of Supports 296
8.5.2 Errors of Fabrication 300
8.6 Analysis of Symmetrical Structures 302
8.6.1 Symmetrical and Antisymmetrical Loading 302
8.6.2 Concept of Half-Structure 303
Problems 305

9 Mixed Method 313
9.1 Fundamental Idea of the Mixed Method 313
9.1.1 Mixed Indeterminacy and Primary Unknowns 313
9.1.2 Primary System 314
9.2 Canonical Equations of the Mixed Method 316
9.2.1 The Matter of Unit Coefficients and Canonical Equations 316

9.2.2 Calculation of Coefficients and Free Terms 317
9.2.3 Computation of Internal Forces 318
Problems 319

10 Influence Lines Method 323
10.1 Construction of Influence Lines by the Force Method 323
10.1.1 Continuous Beams 325
10.1.2 Hingeless Nonuniform Arches 331
10.1.3 Statically Indeterminate Trusses 339
10.2 Construction of Influence Lines by the Displacement Method 344
10.2.1 Continuous Beams 346
10.2.2 Redundant Frames 353
10.3 Comparison of the Force and Displacements Methods 355
10.4 Kinematical Method for Construction of Influence Lines 358
Problems 364

11 Matrix Stiffness Method 369
11.1 Basic Idea and Concepts 369
11.1.1 Finite Elements 370
11.1.2 Global and Local Coordinate Systems 370
11.1.3 Displacements of Joints and Degrees of Freedom 371
11.2 Ancillary Diagrams 372
11.2.1 Joint-Load (J-L) Diagram 372
11.2.2 Displacement-Load (Z-P) Diagram 376
11.2.3 Internal Forces-Deformation (S-e) Diagram 377
11.3 Initial Matrices 379
11.3.1 Vector of External Joint Loads 379
11.3.2 Vector of Internal Unknown Forces 380
11.4 Resolving Equations 381
11.4.1 Static Equations and Static Matrix 381
11.4.2 Geometrical Equations and Deformation Matrix 386
11.4.3 Physical Equations and Stiffness Matrix in Local Coordinates 387
11.5 Set of Formulas and Procedure for Analysis 390
11.5.1 Stiffness Matrix in Global Coordinates 390
11.5.2 Unknown Displacements and Internal Forces 391
11.5.3 Matrix Procedures 392

11.6 Analysis of Continuous Beams 393
11.7 Analysis of Redundant Frames 404
11.8 Analysis of Statically Indeterminate Trusses 410
11.9 Summary 414
Problems 415

Part III Special Topics

12 Plastic Behavior of Structures 423
12.1 Idealized Stress–Strain Diagrams 423
12.2 Direct Method of Plastic Analysis 427
12.3 Fundamental Methods of Plastic Analysis 430
12.3.1 Kinematical Method 430
12.3.2 Static Method 430
12.4 Limit Plastic Analysis of Continuous Beams 432
12.4.1 Static Method 433
12.4.2 Kinematical Method 435
12.5 Limit Plastic Analysis of Frames 441
12.5.1 Beam Failure 442
12.5.2 Sidesway Failure 444
12.5.3 Combined Failure 444
12.5.4 Limit Combination Diagram 444
Problems 445

13 Stability of Elastic Systems 449
13.1 Fundamental Concepts 449
13.2 Stability of Structures with Finite Number Degrees of Freedom 453
13.2.1 Structures with One Degree of Freedom 453
13.2.2 Structures with Two or More Degrees of Freedom 458
13.3 Stability of Columns with Rigid and Elastic Supports 461
13.3.1 The Double Integration Method 461
13.3.2 Initial Parameters Method 466
13.4 Stability of Continuous Beams and Frames 471
13.4.1 Unit Reactions of the Beam-Columns 471
13.4.2 Displacement Method 473
13.4.3 Modified Approach of the Displacement Method 481
13.5 Stability of Arches 483
13.5.1 Circular Arches Under Hydrostatic Load 484
13.5.2 Complex Arched Structure: Arch with Elastic Supports 490

13.6 Compressed Rods with Lateral Loading 491
13.6.1 Double Integration Method 492
13.6.2 Initial Parameters Method 494
13.6.3 P-Delta Analysis of the Frames 499
13.6.4 Graph Multiplication Method for Beam-Columns 502
Problems 504

14 Dynamics of Elastic Systems 513
14.1 Fundamental Concepts 513
14.1.1 Kinematics of Vibrating Processes 513
14.1.2 Forces Which Arise at Vibrations 513
14.1.3 Degrees of Freedom 515
14.1.4 Purpose of Structural Dynamics 519
14.1.5 Assumptions 519
14.2 Free Vibrations of Systems with Finite Number Degrees of Freedom: Force Method 520
14.2.1 Differential Equations of Free Vibration in Displacements 520
14.2.2 Frequency Equation 521
14.2.3 Mode Shapes Vibration and Modal Matrix 522
14.3 Free Vibrations of Systems with Finite Number Degrees of Freedom: Displacement Method 530
14.3.1 Differential Equations of Free Vibration in Reactions 530
14.3.2 Frequency Equation 532
14.3.3 Mode Shape Vibrations and Modal Matrix 532
14.3.4 Comparison of the Force and Displacement Methods 538
14.4 Free Vibrations of One-Span Beams with Uniformly Distributed Mass 538
14.4.1 Differential Equation of Transversal Vibration of the Beam 540
14.4.2 Fourier Method 541
14.4.3 Krylov–Duncan Method 543
Problems 546

Appendix 551

Bibliography 587

Index 589