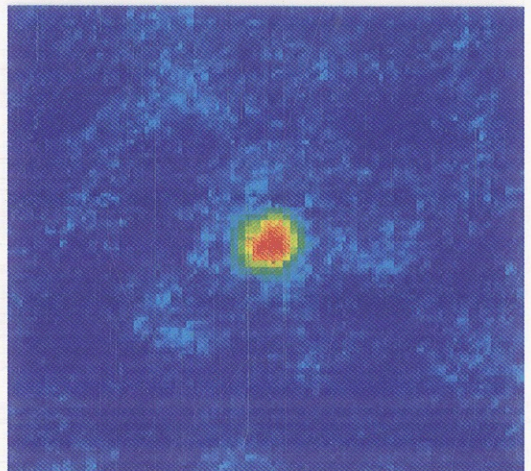
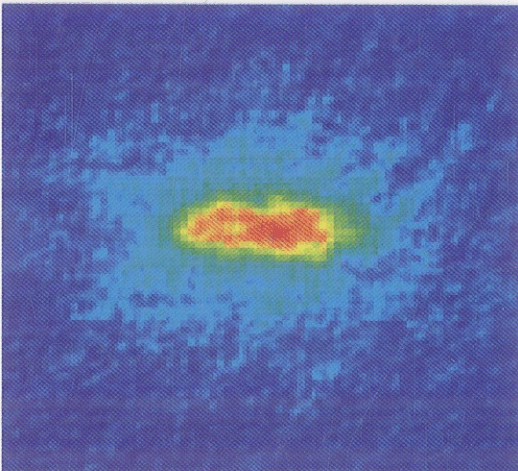
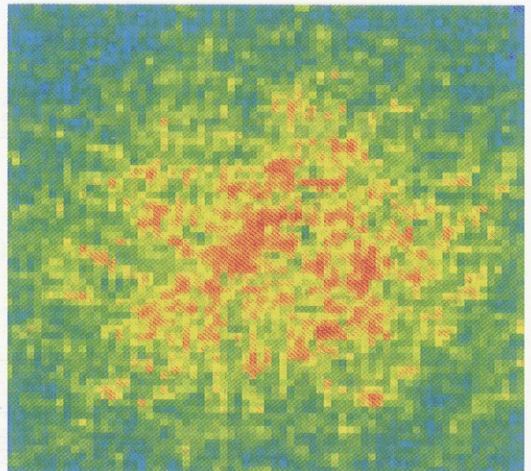
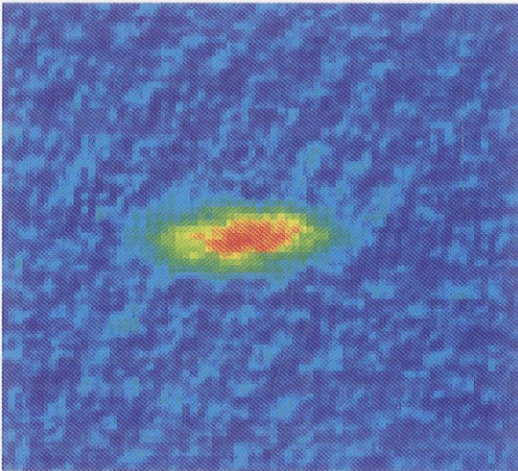


Linda E. Reichl

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# A Modern Course in Statistical Physics

Third Revised and Updated Edition



# Contents

**Preface to the Third Edition** XIII

**Preface to the First Edition** XV

- 1 Introduction** 1
- 2 Complexity and Entropy** 5
  - 2.1 Introduction 5
  - 2.2 Counting Microscopic States 5
  - 2.3 Multiplicity and Entropy of Macroscopic Physical States 9
  - 2.4 Multiplicity and Entropy of a Spin System 10
    - 2.4.1 Multiplicity of a Spin System 10
    - 2.4.2 Entropy of Spin System 11
      - 2.4.2.1 Entropy and Fluctuations About Equilibrium 12
      - 2.4.2.2 Entropy and Temperature 12
  - 2.5 Multiplicity and Entropy of an Einstein Solid 14
    - 2.5.1 Multiplicity of an Einstein Solid 15
    - 2.5.2 Entropy of the Einstein Solid 15
  - 2.6 Multiplicity and Entropy of an Ideal Gas 16
    - 2.6.1 Multiplicity of an Ideal Gas 17
    - 2.6.2 Entropy of an Ideal Gas 18
  - 2.7 Problems 19
- 3 Thermodynamics** 21
  - 3.1 Introduction 21
  - 3.2 Energy Conservation 23
  - 3.3 Entropy 24
    - 3.3.1 Carnot Engine 24
    - 3.3.2 The Third Law 28
  - 3.4 Fundamental Equation of Thermodynamics 29
  - 3.5 Thermodynamic Potentials 32
    - 3.5.1 Internal Energy 33
    - 3.5.2 Enthalpy 34
    - 3.5.3 Helmholtz Free Energy 35

3.5.4	Gibbs Free Energy	37
3.5.5	Grand Potential	38
3.6	Response Functions	40
3.6.1	Thermal Response Functions (Heat Capacity)	40
3.6.2	Mechanical Response Functions	42
3.7	Stability of the Equilibrium State	45
3.7.1	Conditions for Local Equilibrium in a PVT System	45
3.7.2	Conditions for Local Stability in a PVT System	46
3.7.3	Implications of the Stability Requirements for the Free Energies	50
3.7.4	Correlations Between Fluctuations	52
3.8	Cooling and Liquefaction of Gases	55
3.9	Osmotic Pressure in Dilute Solutions	58
3.10	The Thermodynamics of Chemical Reactions	61
3.10.1	The Affinity	62
3.11	Problems	67
<b>4</b>	<b>The Thermodynamics of Phase Transitions</b>	<b>75</b>
4.1	Introduction	75
4.2	Coexistence of Phases: Gibbs Phase Rule	76
4.3	Classification of Phase Transitions	77
4.4	Classical Pure PVT Systems	79
4.4.1	Phase Diagrams	79
4.4.2	Coexistence Curves: Clausius–Clapeyron Equation	80
4.4.3	Liquid–Vapor Coexistence Region	83
4.4.3.1	Lever Rule	84
4.4.3.2	Law of Corresponding States	84
4.4.3.3	Response Functions in the Coexistence Region	85
4.4.4	The van der Waals Equation	87
4.4.5	Steam Engines – The Rankine Cycle	90
4.5	Binary Mixtures	93
4.5.1	Equilibrium Conditions	94
4.6	The Helium Liquids	96
4.6.1	Liquid He <sup>4</sup>	97
4.6.1.1	Thermomechanical Effect	97
4.6.2	Liquid He <sup>3</sup>	99
4.6.3	Liquid He <sup>3</sup> –He <sup>4</sup> Mixtures	100
4.7	Superconductors	101
4.8	Ginzburg–Landau Theory	104
4.8.1	Theoretical Background	105
4.8.2	Applications of Ginzburg–Landau Theory	108
4.8.2.1	Superfluids	108
4.8.2.2	The Curie Point	108
4.8.2.3	Superconductors	109
4.9	Critical Exponents	110

4.9.1	Definition of Critical Exponents	110
4.9.2	The Critical Exponents for Pure <i>PVT</i> Systems	111
4.9.3	The Critical Exponents for the Curie Point	113
4.9.4	The Critical Exponents for Mean Field Theories	114
4.10	Problems	116
<b>5</b>	<b>Equilibrium Statistical Mechanics i – Canonical Ensemble</b>	<b>121</b>
5.1	Introduction	121
5.2	Probability Density Operator-Canonical Ensemble	123
5.2.1	Energy Fluctuations	124
5.3	Semiclassical Ideal Gas of Indistinguishable Particles	125
5.3.1	Approximations to the Partition Function for Semiclassical Ideal Gases	126
5.3.2	Maxwell–Boltzmann Distribution	129
5.4	Interacting Classical Fluids	131
5.4.1	Density Correlations and the Radial Distribution Function	132
5.4.2	Magnetization Density Correlations	134
5.5	Heat Capacity of a Debye Solid	135
5.6	Order–Disorder Transitions on Spin Lattices	139
5.6.1	Exact Solution for a One-Dimensional Lattice	140
5.6.2	Mean Field Theory for a <i>d</i> -Dimensional Lattice	142
5.6.3	Mean Field Theory of Spatial Correlation Functions	145
5.6.4	Exact Solution to Ising Lattice for <i>d</i> = 2	146
5.7	Scaling	148
5.7.1	Homogeneous Functions	148
5.7.2	Widom Scaling	149
5.7.3	Kadanoff Scaling	152
5.8	Microscopic Calculation of Critical Exponents	155
5.8.1	General Theory	156
5.8.2	Application to Triangular Lattice	158
5.8.3	The $S^4$ Model	161
5.9	Problems	163
<b>6</b>	<b>Equilibrium Statistical Mechanics ii – Grand Canonical Ensemble</b>	<b>167</b>
6.1	Introduction	167
6.2	The Grand Canonical Ensemble	168
6.2.1	Particle Number Fluctuations	169
6.2.2	Ideal Classical Gas	170
6.3	Virial Expansion for Interacting Classical Fluids	172
6.3.1	Virial Expansion and Cluster Functions	172
6.3.2	The Second Virial Coefficient, $B_2(T)$	174
6.3.2.1	Square-Well Potential	175
6.3.2.2	Lennard-Jones 6–12 Potential	176
6.4	Black Body Radiation	178
6.5	Ideal Quantum Gases	181

6.6	Ideal Bose–Einstein Gas	183
6.6.1	Bose–Einstein Condensation	187
6.6.2	Experimental Observation of Bose–Einstein Condensation	189
6.7	Ideal Fermi–Dirac Gas	191
6.8	Momentum Condensation in an Interacting Fermi Fluid	197
6.9	Problems	204
<b>7</b>	<b>Brownian Motion and Fluctuation–Dissipation</b>	<b>207</b>
7.1	Introduction	207
7.2	Brownian Motion	208
7.2.1	Langevin Equation	209
7.2.2	Correlation Function and Spectral Density	210
7.3	The Fokker–Planck Equation	212
7.3.1	Probability Flow in Phase Space	214
7.3.2	Probability Flow for Brownian Particle	214
7.3.3	The Strong Friction Limit	217
7.3.3.1	Spectral Decomposition of the Fokker–Planck Equation	218
7.4	Dynamic Equilibrium Fluctuations	221
7.4.1	Regression of Fluctuations	223
7.4.2	Wiener–Khinchine Theorem	225
7.5	Linear Response Theory and the Fluctuation – Dissipation Theorem	226
7.5.1	Linear Response Theory and the Fluctuation – Dissipation Theorem	227
7.5.2	Causality	228
7.5.2.1	Piece-Wise Constant Force	230
7.5.3	The Fluctuation–Dissipation Theorem	231
7.5.4	Power Absorption	233
7.5.4.1	Delta Function Force	234
7.5.4.2	Oscillating Force	234
7.6	Microscopic Linear Response Theory	235
7.6.1	The Perturbed Density Operator	235
7.6.2	The Electric Conductance	236
7.6.3	Power Absorption	241
7.6.4	Thermal Noise	242
7.7	Problems	243
<b>8</b>	<b>Hydrodynamics</b>	<b>247</b>
8.1	Introduction	247
8.2	Navier–Stokes Hydrodynamic Equations	248
8.2.1	Balance Equations	248
8.2.1.1	Mass Density Equations	250
8.2.1.2	Momentum Balance Equation	250
8.2.1.3	Energy and Entropy Balance Equations	252
8.2.2	Entropy Source and Entropy Current	253
8.2.3	Transport Coefficients	257

8.3	Linearized Hydrodynamic Equations	260
8.3.1	Linearization of the Hydrodynamic Equations	260
8.3.2	Transverse Hydrodynamic Modes	264
8.3.3	Longitudinal Hydrodynamic Modes	265
8.3.4	Dynamic Correlation Function and Spectral Density	267
8.4	Light Scattering	268
8.4.1	Scattered Electric Field	270
8.4.2	Intensity of Scattered Light	272
8.5	Hydrodynamics of Mixtures	274
8.5.1	Entropy Production in Multicomponent Systems	275
8.5.2	Fick's Law for Diffusion	277
8.5.3	Thermal Diffusion	279
8.6	Thermoelectricity	280
8.6.1	The Peltier Effect	280
8.6.2	The Seebeck Effect	282
8.6.3	Thomson Heat	284
8.7	Superfluid Hydrodynamics	284
8.7.1	Superfluid Hydrodynamic Equations	284
8.7.2	Sound Modes	288
8.8	Problems	291
<b>9</b>	<b>Transport Coefficients</b>	<b>295</b>
9.1	Introduction	295
9.2	Elementary Transport Theory	295
9.2.1	The Mean Free Path	296
9.2.2	The Collision Frequency	296
9.2.3	Tracer Particle Current	298
9.2.4	Transport of Molecular Properties	300
9.2.5	The Rate of Reaction	301
9.3	The Boltzmann Equation	303
9.3.1	Derivation of the Boltzmann Equation	304
9.4	Linearized Boltzmann Equation	304
9.4.1	Kinetic Equations for a Two-Component Gas	305
9.4.2	Collision Operators	306
9.5	Coefficient of Self-Diffusion	308
9.5.1	Derivation of the Diffusion Equation	308
9.5.2	Eigenfrequencies of the Lorentz–Boltzmann Equation	309
9.6	Coefficients of Viscosity and Thermal Conductivity	311
9.6.1	Derivation of the Hydrodynamic Equations	311
9.6.2	Eigenfrequencies of the Boltzmann Equation	315
9.6.3	Shear Viscosity and Thermal Conductivity	317
9.7	Computation of Transport Coefficients	318
9.7.1	Sonine Polynomials	319
9.7.2	Diffusion Coefficient	319
9.7.3	Thermal Conductivity	321

9.7.4	Shear Viscosity	323
9.8	Problems	324
<b>10</b>	<b>Nonequilibrium Phase Transitions</b>	<b>327</b>
10.1	Introduction	327
10.2	Near Equilibrium Stability Criteria	328
10.3	The Chemically Reacting Systems	330
10.3.1	The Brusselator – A Nonlinear Chemical Model	330
10.3.2	Boundary Conditions	332
10.3.2.1	Case I Boundary Conditions	332
10.3.2.2	Case II Boundary Conditions	332
10.3.3	Stability Analysis	333
10.3.3.1	Real Frequency $\omega(k)$	334
10.3.3.2	Complex Frequency $\omega(k)$	334
10.3.4	Chemical Crystals	335
10.4	The Rayleigh–Bénard Instability	337
10.4.1	Hydrodynamic Equations and Boundary Conditions	337
10.4.1.1	Rigid Surface Boundary Conditions	339
10.4.1.2	Smooth Surface Boundary Conditions	339
10.4.2	Linear Stability Analysis	340
10.5	Problems	343
<b>Appendix A</b>	<b>Probability</b>	<b>345</b>
A.1	Definition of Probability	345
A.2	Probability Distribution Functions	347
A.2.1	Discrete Stochastic Variables	348
A.2.2	Continuous Stochastic Variables	348
A.2.3	Characteristic Function	349
A.2.4	Jointly Distributed Stochastic Variables	350
A.3	Binomial Distributions	351
A.3.1	The Binomial Distribution	351
A.3.2	The Gaussian (or Normal) Distribution	353
A.3.3	The Poisson Distribution	356
A.4	Markov Chains	358
A.5	Probability Density for Classical Phase Space	359
A.6	Quantum Probability Density Operator	363
A.7	Problems	366
<b>Appendix B</b>	<b>Exact Differentials</b>	<b>369</b>
<b>Appendix C</b>	<b>Ergodicity</b>	<b>373</b>
<b>Appendix D</b>	<b>Number Representation</b>	<b>377</b>
D.1	The Number Representation	383
D.1.1	The Number Representation for Bosons	383
D.1.2	The Number Representation for Fermions	385
D.1.3	Thermodynamic Averages of Quantum Operators	387

<b>Appendix E Scattering Theory</b>	389
<b>Appendix F Useful Mathematics and Information</b>	395
F.1 Series Expansions	395
F.2 Reversion of Series	395
F.3 Derivatives	395
F.4 Integrals	395
<b>References</b>	397
<b>Index</b>	403