FINITE ELEMENT ANALYSIS AND PROGRAMMING

An Introduction

Shashishekar Shivaswamy

Alpha Science

TABLE OF CONTENTS

Preface		*vii*
List of Abbreviations and Symbols		*ix*
CHAPTER 1. WHAT IS FEA?		**1.1**
1.0	Introduction	1.1
1.1	FEA procedure	1.1
1.2	How to use this book	1.2
CHAPTER 2. BASIC CONCEPTS IN FEA		**2.1**
2.0	Introduction	2.1
2.1	Stress	2.1
2.2	Strain	2.2
2.3	Stress-strain relationship in 3 dimensions	2.4
2.4	Stress-strain relationships in 2 dimensions	2.5
	2.4.1 Plane stress condition	2.6
	2.4.2 Plane strain condition	2.6
	2.4.3 Axisymmetric condition	2.7
2.5	Stress-strain relationship in 1 dimension	2.8
2.6	Principal stresses and von Mises stress	2.9
2.7	Strain energy	2.9
2.8	Potential energy	2.10
2.9	Rayleigh-Ritz method	2.11
2.10	Principle of minimum potential energy	2.12
2.11	Principle of virtual work	2.13
2.12	Weak formulation	2.14
2.13	Exercises	2.20
CHAPTER 3. BASIC STEPS IN FEA		**3.1**
3.0	Introduction	3.1
3.1	Formation of element stiffness matrix	3.1
	3.1.1 Gauss-Legendre numerical integration	3.3
3.2	Global stiffness matrix assembly	3.10
3.3	Application of geometric boundary conditions	3.14
3.4	Formation of the consistent load vector	3.22

3.5 Solving for displacement 3.24
3.5.1 Pivoting in Gaussian elimination 3.26
3.6 Calculation of derived parameters 3.29
3.7 Exercises 3.30

CHAPTER 4. DISPLACEMENT FUNCTION BASED FORMULATION 4.1
4.0 Introduction 4.1
4.1 Displacement based finite element formulation 4.1
4.1.1 Compatibility 4.1
4.1.2 Representation of rigid body modes 4.2
4.1.3 No preferred directions 4.2
4.1.4 Ability to produce constant stress when subjected to state of constant strain 4.3
4.1.5 Convergence 4.3
4.2 Displacement field for one dimensional elements 4.4
4.3 Displacement fields for two dimensional membrane elements 4.5
4.4 Displacement fields for three dimensional solid elements 4.7
4.5 Displacement field for plate elements 4.9
4.6 Exercises 4.9

CHAPTER 5. SHAPE FUNCTIONS 5.1
5.0 Introduction 5.1
5.1 One dimensional Lagrange shape functions 5.1
5.2 Two dimensional shape functions 5.3
5.2.1 Triangle elements 5.3
5.2.1.a Linear shape functions over a three noded triangle 5.3
5.2.1.b Quadratic shape functions over a six noded triangle 5.7
5.2.2 Quadrilateral elements 5.8
5.2.2.a Four noded quadrilateral element 5.9
5.2.2.b Higher order serendipity quadrilateral elements 5.10
5.2.2.c Lagrange quadrilateral elements 5.13
5.2.2.d Hermitian quadrilateral element 5.16
5.3 Three dimensional shape functions 5.18
5.3.1 Tetrahedron element 5.18
5.3.2 Prism elements 5.20
5.3.3 Hexahedron elements 5.21
5.4 Concept of Jacobian matrix 5.22
5.5 Exercises 5.24

CHAPTER 6. BAR ELEMENTS 6.1
6.0 Introduction 6.1
6.1 Linear bar element 6.1
6.2 Higher order bar element 6.3
6.3 Bar element spreadsheet program 6.13
6.4 Exercises 6.19

CHAPTER 7. TRUSS ELEMENTS **7.1**
7.0 Introduction 7.1
7.1 Two dimensional truss 7.1
7.2 Three dimensional truss 7.3
7.3 Truss element spreadsheet program 7.11
7.4 Exercises 7.14

CHAPTER 8. BEAM ELEMENTS **8.1**
8.0 Introduction 8.1
8.1 Euler beam 8.1
8.2 Euler beam element spreadsheet program 8.10
8.3 Transverse shear deformation in beams 8.12
8.4 Timoshenko beam spreadsheet program 8.23
8.5 Axial deformation in beams 8.26
8.6 Torsional deformation in beams 8.26
8.7 Out of plane bending in beams 8.26
8.8 Two and three dimensional beam elements 8.27
8.9 Offset beam element 8.29
8.10 Exercises 8.29

CHAPTER 9. FRAME ELEMENT **9.1**
9.0 Introduction 9.1
9.1 Two dimensional frame 9.1
9.2 Two dimensional frame spreadsheet program 9.11
9.3 Three dimensional frame 9.14
9.4 Three dimensional rigid element 9.16
9.5 Exercises 9.16

CHAPTER 10. STRAIN-DISPLACEMENT MATRIX FOR 2D AND 3D ELEMENTS **10.1**
10.0 Introduction 10.1
10.1 Displacement function based approach 10.1
10.2 Isoparametric formulation 10.3

CHAPTER 11. TRIANGLE MEMBRANE ELEMENTS **11.1**
11.0 Introduction 11.1
11.1 Three noded triangle membrane element (TRIA3M) 11.1
11.2 CST element spreadsheet program 11.6
11.3 Six noded triangle membrane element (TRIA6M) 11.9
11.4 TRIA6M element in fracture mechanics 11.23
11.5 Exercises 11.23

CHAPTER 12. QUADRILATERAL MEMBRANE ELEMENTS **12.1**
12.0 Introduction 12.1
12.1 Displacement function based formulation 12.1
12.2 Four noded quadrilateral membrane element (QUAD4M) 12.6
12.3 QUAD4M element spreadsheet program 12.21

12.4 Eight noded quadrilateral membrane element (QUAD8M) 12.25
12.5 Order of integration 12.27
12.6 Parasitic shear due to bending 12.34
12.7 Connecting a membrane element to a 2D beam element 12.36
12.8 Exercises 12.36

CHAPTER 13. AXISYMMETRIC ELEMENTS **13.1**
13.0 Introduction 13.1
13.1 Three noded triangle axisymmetric element (TRIA3A) 13.1
13.2 Four noded quadrilateral axisymmetric element (QUAD4A) 13.3
13.3 Consistent load vector 13.4
13.4 Exercises 13.20

CHAPTER 14. TETRAHEDRON ELEMENTS **14.1**
14.0 Introduction 14.1
14.1 Four noded tetrahedron element (TET4) 14.1
14.2 Ten noded tetrahedron element (TET10) 14.22
14.3 Exercises 14.24

CHAPTER 15. PRISM ELEMENT **15.1**
15.0 Introduction 15.1
15.1 Six noded prism element (PRISM6) 15.1
15.2 Exercises 15.21

CHAPTER 16. HEXAHEDRON ELEMENT **16.1**
16.0 Introduction 16.1
16.1 Eight noded hexahedron element (HEXA8) 16.1
16.2 Exercises 16.19

CHAPTER 17. PLATE ELEMENTS **17.1**
17.0 Introduction 17.1
17.1 Thin and thick plates 17.1
17.2 Basic plate equations 17.1
17.3 Strain energy 17.4
17.4 Continuity requirements 17.5
17.5 FE formulation of thin plates 17.5
17.5.1 Displacement function based formulation of a three noded triangle plate element (TRIA3P) 17.5
17.5.2 Displacement function based formulation of a 4 noded rectangle plate element 17.10
17.6 Reissner-Mindlin plate theory 17.12
17.6.1 Four noded isoparametric thick plate element 17.14
17.7 Flat shell element 17.32
17.8 Exercises 17.33

CHAPTER 18. COMPOSITE LAMINATES **18.1**
18.0 Introduction 18.1
18.1 Orthotropic material stress-strain relationship about principal axes 18.1

18.2 General stress-strain relationship of an orthotropic lamina 18.2
18.3 Laminate plate 18.3
18.4 Classical laminate plate theory (CLPT) 18.5
18.5 First order shear deformation laminate plate theory (FSDT) 18.6
18.6 Finite element formulation of CLPT 18.7
18.7 Finite element formulation of FSDT 18.8
18.8 Exercises 18.32

CHAPTER 19. DYNAMIC ANALYSIS **19.1**
19.0 Introduction 19.1
19.1 Lagrange's equation 19.1
19.2 Eigenvalue problem 19.3
19.3 Properties of eigenvectors 19.5
19.4 Lagrange's equation for non-conservative systems 19.7
19.5 Numerical integration of equations of motion 19.17
19.5.1 Runge-Kutta fourth order method 19.17
19.5.2 Crank-Nicolson method 19.18
19.5.3 Wilson θ method 19.19
19.5.4 Newmark method 19.20
19.6 Application of Lagrange's equation in FEA 19.30
19.6.1 Case (i) 19.30
19.6.1.a Two noded bar element 19.32
19.6.1.b Truss element 19.32
19.6.1.c Triangle membrane element 19.33
19.6.1.d Rectangle and quadrilateral membrane element 19.33
19.6.1.e Tetrahedron element 19.34
19.6.1.f Prism element 19.35
19.6.1.g Hexahedron elements 19.36
19.6.2 Case (ii) 19.37
19.6.2.a Beam and frame element based on Euler theory 19.37
19.6.2.b Plate elements based on Kirchoff–Love theory 19.38
19.6.3 Case (iii) 19.39
19.6.3.a Timoshenko beam element 19.39
19.6.3.b Reissner-Mindlin plate element 19.40
19.7 Application of boundary conditions in dynamic FEA 19.44
19.8 Exercises 19.67

CHAPTER 20. FEA OF PARTIAL DIFFERTIAL EQUATIONS **20.1**
20.0 Introduction 20.1
20.1 Elliptical system 20.1
20.1.1 Elliptical system – 1D 20.1
20.1.2 Elliptical system – 2D 20.14
20.1.2.a Inviscid incompressible fluid flow 20.19
20.1.2.b Torsional deformation of a prismatic bar 20.20
20.1.2.c Helmoltz' equation 20.40
20.1.3 Elliptical system – 3D 20.40

20.2 Parabolic system 20.42
20.2.1 Parabolic system - 1D 20.42
20.3 Hyperbolic system 20.48
20.3.1 Hyperbolic system – 1D 20.48
20.4 Exercises 20.54

APPENDIX A. METHODS FOR SOLVING A SYSTEM OF EQUATIONS A.1
A.0 Introduction A.1
A.1 LU factorization A.1
A.2 Cholesky's method A.5

APPENDIX B. METHODS FOR SOLVING EIGENVALUE PROBLEMS B.1
B.0 Introduction B.1
B.1 Eigenvalue problem in standard form B.1
B.1.1 Power method B.1
B.1.2 Shifted iteration B.2
B.1.3 Shifted inverse iteration B.2
B.1.4 Deflation technique B.2
B.1.5 Jacobi transformation B.3
B.1.6 Givens method B.4
B.2 Eigenvalue problem in general form B.5
B.2.1 Power method B.6
B.2.2 Rayleigh quotient B.6
B.2.3 Generalized Jacobi method B.6

APPENDIX C. PROGRAMMING GUIDE TO MATHCAD 2001 C.1
C.0 Introduction C.1
C.1 Other versions of MATHCAD C.9

APPENDIX D. UNIT CONVERSIONS D.1

References R.1

Subject Index S.1