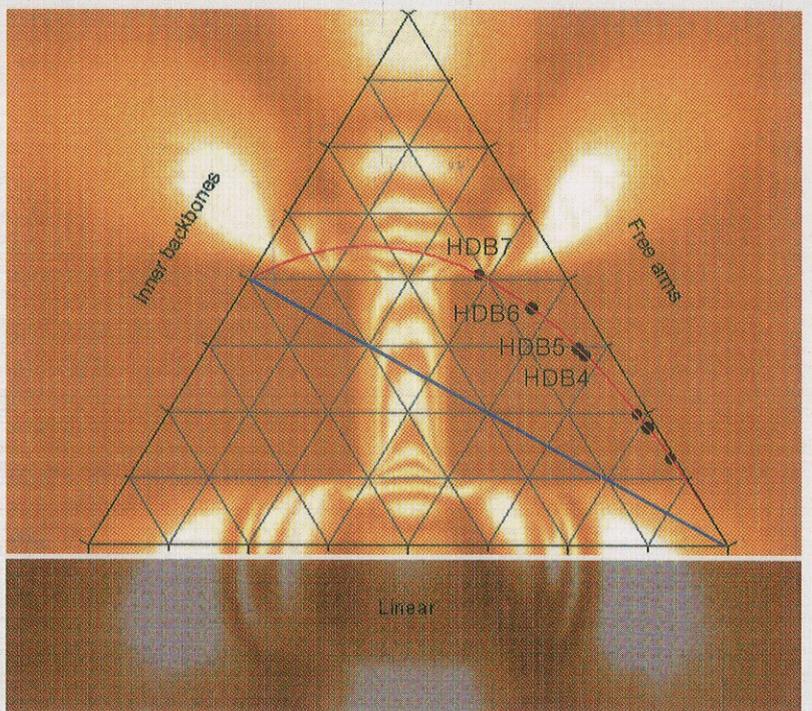


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# Structure and Rheology of Molten Polymers

From Structure to Flow Behavior  
and Back Again



HANSER

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