

Fundamentals *of* Matrix Computations



CONTENTS

Preface	ix
Acknowledgments	xv
1 Gaussian Elimination and Its Variants	1
1.1 Matrix Multiplication	1
1.2 Systems of Linear Equations	12
1.3 Triangular Systems	24
1.4 Positive Definite Systems; Cholesky Decomposition	33
1.5 Banded Positive Definite Systems	55
1.6 Sparse Positive Definite Systems	64
1.7 Gaussian Elimination and the LU Decomposition	71
1.8 Gaussian Elimination with Pivoting	94
1.9 Sparse Gaussian Elimination	107
2 Sensitivity of Linear Systems	113
2.1 Vector and Matrix Norms	114
2.2 Condition Numbers	122

2.3	Perturbing the Coefficient Matrix	133
2.4	A Posteriori Error Analysis Using the Residual	137
2.5	Roundoff Errors; Backward Stability	139
2.6	Propagation of Roundoff Errors	149
2.7	Backward Error Analysis of Gaussian Elimination	157
2.8	Scaling	171
2.9	Componentwise Sensitivity Analysis	176
3	The Least Squares Problem	183
3.1	The Discrete Least Squares Problem	183
3.2	Orthogonal Matrices, Rotators, and Reflectors	187
3.3	Solution of the Least Squares Problem	215
3.4	The Gram-Schmidt Process	223
3.5	Geometric Approach	238
3.6	Updating the QR Decomposition	247
4	The Singular Value Decomposition	259
4.1	Introduction	260
4.2	Some Basic Applications of Singular Values	264
4.3	The SVD and the Least Squares Problem	273
4.4	Sensitivity of the Least Squares Problem	279
5	Eigenvalues and Eigenvectors I	289
5.1	Systems of Differential Equations	289
5.2	Basic Facts	305
5.3	The Power Method and Some Simple Extensions	314
5.4	Similarity Transforms	334
5.5	Reduction to Hessenberg and Tridiagonal Forms	350
5.6	Francis's Algorithm	358
5.7	Use of Francis's Algorithm to Calculate Eigenvectors	386
5.8	The SVD Revisited	389
6	Eigenvalues and Eigenvectors II	409
6.1	Eigenspaces and Invariant Subspaces	410
6.2	Subspace Iteration and Simultaneous Iteration	420
6.3	Krylov Subspaces and Francis's Algorithm	428
6.4	Large Sparse Eigenvalue Problems	437

6.5	Implicit Restarts	456
6.6	The Jacobi-Davidson and Related Algorithms	466
7	Eigenvalues and Eigenvectors III	471
7.1	Sensitivity of Eigenvalues and Eigenvectors	471
7.2	Methods for the Symmetric Eigenvalue Problem	485
7.3	Product Eigenvalue Problems	511
7.4	The Generalized Eigenvalue Problem	526
8	Iterative Methods for Linear Systems	545
8.1	A Model Problem	546
8.2	The Classical Iterative Methods	554
8.3	Convergence of Iterative Methods	568
8.4	Descent Methods; Steepest Descent	583
8.5	On Stopping Criteria	594
8.6	Preconditioners	596
8.7	The Conjugate-Gradient Method	602
8.8	Derivation of the CG Algorithm	607
8.9	Convergence of the CG Algorithm	615
8.10	Indefinite and Nonsymmetric Problems	621
	References	627
	Index	635
	Index of MATLAB[®] Terms	643