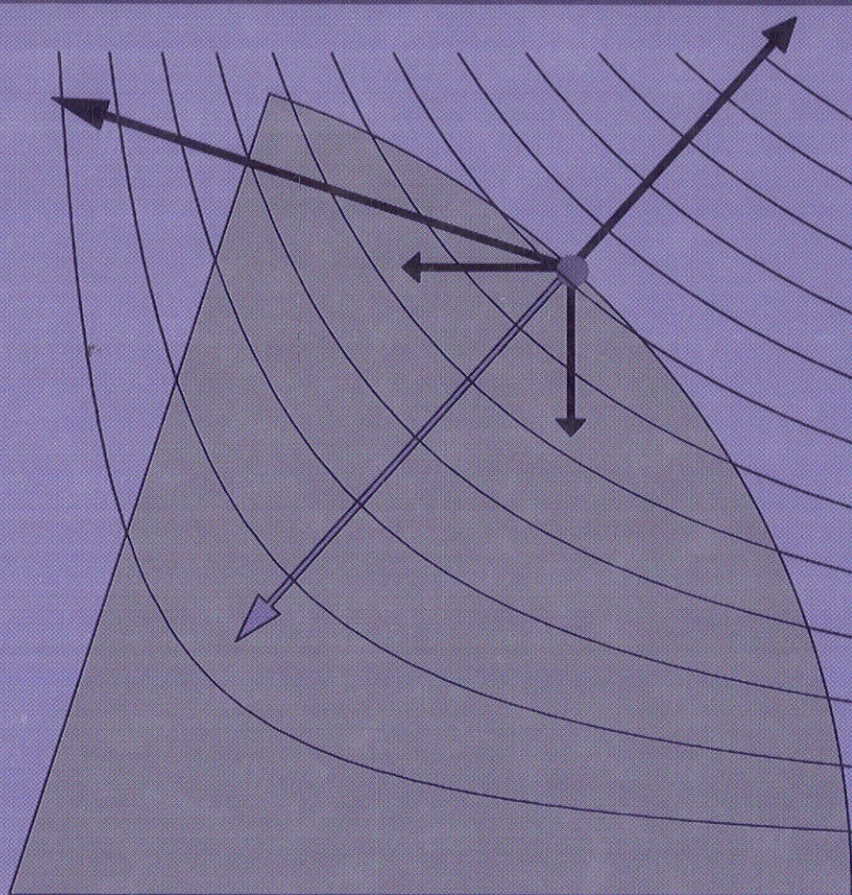


TEXTBOOKS in MATHEMATICS

LINEAR AND NONLINEAR PROGRAMMING WITH MAPLE™

An Interactive,
Applications-Based Approach



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A CHAPMAN & HALL BOOK

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