



CIRCUIT SIMULATION

3V

00000

$1\mu H$

③

$10k\Omega$

FARID N. NAJM

450Ω

②

640Ω

④

$2pF$

⑤

$2pF$

①

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