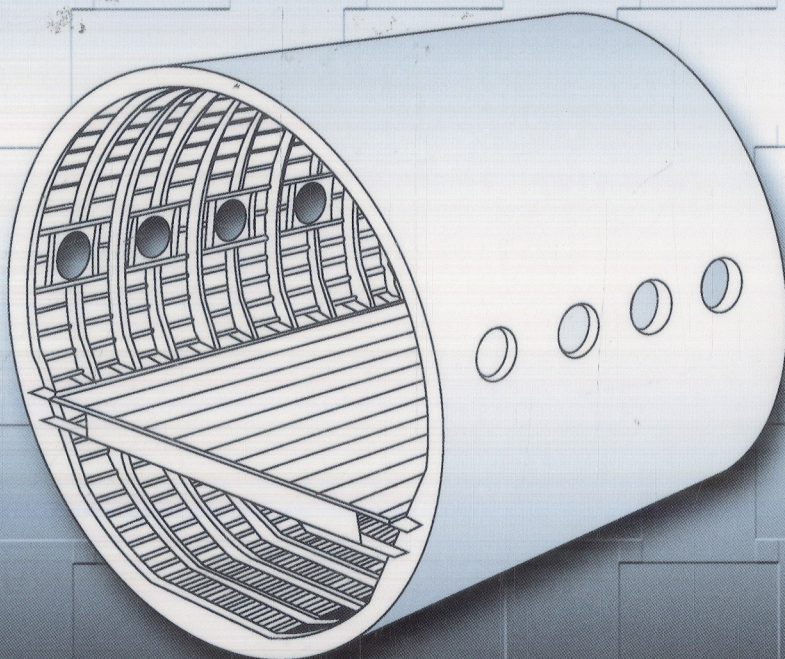


CRC Series in
COMPUTATIONAL MECHANICS and APPLIED ANALYSIS

STRESSES IN BEAMS, PLATES, AND SHELLS

Third Edition



ANSEL C. UGURAL



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