Nonlinear Systems of Partial Differential Equations

Applications to Life and Physical Sciences

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World Scientific

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