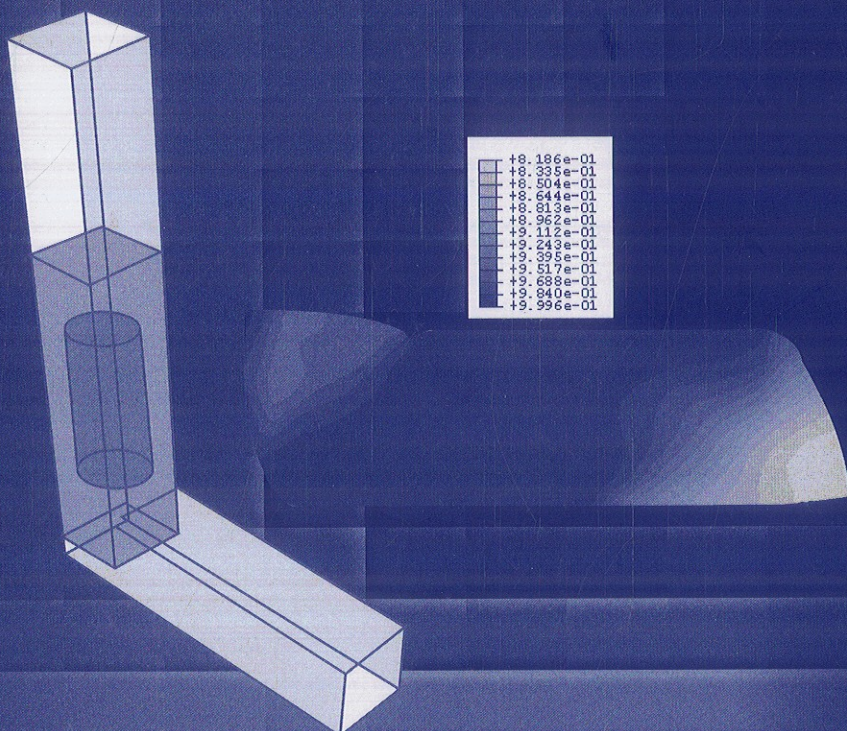


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INELASTICITY OF MATERIALS

An Engineering Approach and a Practical Guide

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