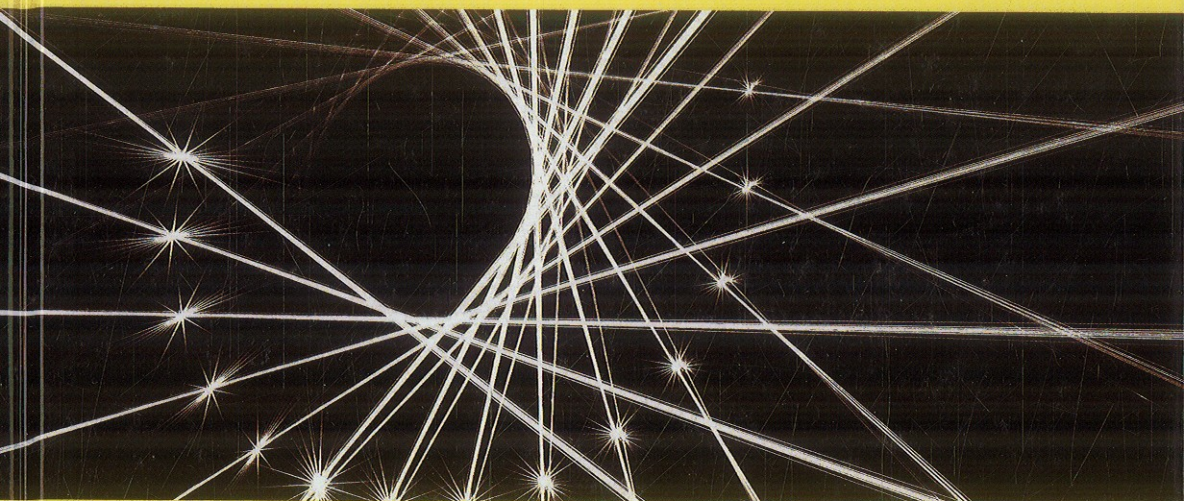


CAM

Control Systems, Robotics and Manufacturing Series



Linear Systems

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ISTE

 **WILEY**

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