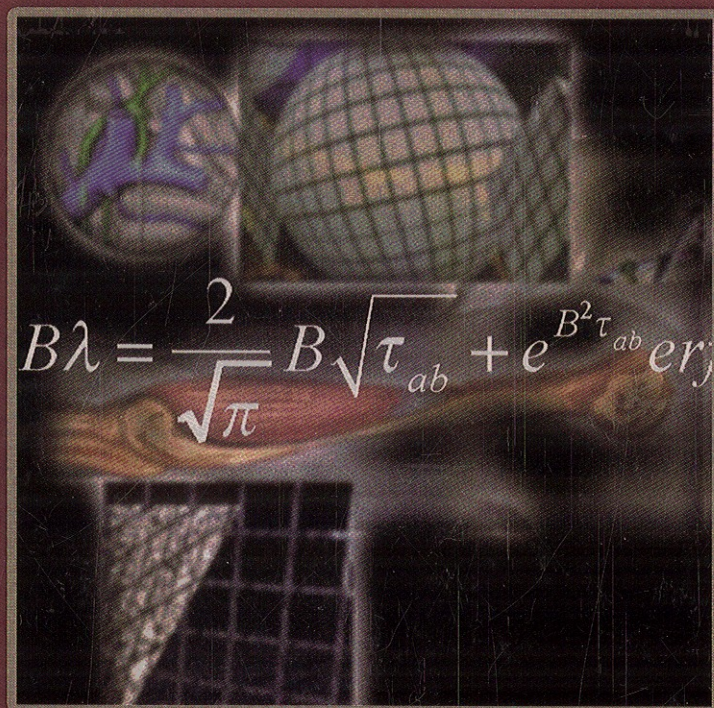


ACADEMIC PRESS SERIES IN BIOMEDICAL ENGINEERING

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# NUMERICAL METHODS IN BIOMEDICAL ENGINEERING

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