Contents

Preface v
Overview vii

Part I CONDUCTION

1. Energy Equation 3

1.1 Reynolds transport theorem 3
1.2 Derivation of energy equation 5
1.2.1 Constitutive equations 8
1.3 Thermal energy equation in a solid medium 10
1.4 Remarks on energy equation 10
1.5 Fourier's law revisited 13
1.6 Thermal conductivity as a tensor 13
1.7 Expanded forms of the energy equation 14
1.7.1 Governing equation in Cartesian coordinates 14
1.7.2 Cylindrical and spherical coordinates 15
1.8 What is thermal conductivity 16
1.9 What is specific heat 18
1.10 Initial and boundary conditions 20
1.10.1 Composite media 23
1.11 Non-dimensionalization 25
1.12 Fin equation 27
1.13 Phase transformation 29
1.14 Closure 33
Bibliography 33
Problems 35

2. Steady Heat Conduction in One Dimension 37

2.1 Slab and circular geometries 37
2.2 Heat conduction in a fin 43
Problems 48

3. Steady Heat Conduction in Two Dimensions **51**

3.1 Heat conduction in a square domain 51
3.1.1 An example with a convective boundary condition 56
3.1.2 An example with contact resistance 59
3.1.3 An insulated bar with heat generation 62
3.2 Heat conduction in cylindrical coordinates 64
3.2.1 Example where boundary conditions are homogeneous in the ϕ-direction 65
3.2.2 Example where boundary conditions are homogeneous in the r-direction 68
3.2.3 An example in r-z coordinates 69
3.2.4 Note on Bessel functions 72
Problems 73

4. Unsteady Heat Conduction in 1 and 2D Regions **77**

4.1 Bounded one dimensional domains 77
4.1.1 Important remarks 80
4.1.2 Slab with heat generation 80
4.1.3 Principle of superposition 81
4.1.4 Temperature dependent thermal conductivity 82
4.1.5 Thermal resistance 84
4.1.6 Cylindrical coordinates 85
4.2 Transient response of linear systems 88
4.2.1 Time-dependent boundary conditions 91
4.2.2 Time-dependent source term 92
4.3 Semi-infinite solid 93
4.3.1 Time-periodic boundary conditions 96
4.4 Two dimensional regions 102
4.4.1 Polar coordinates 104
4.4.2 Time-dependent boundary conditions 105
Problems 108

5. Phase Transformation **115**

5.1 One dimensional phase change 115
5.1.1 Stefan condition 116
5.1.2 Special cases of Stefan condition 117
5.2 Movement of a phase front in a supercooled liquid 119
5.3 Heat transfer in solid and liquid phases 121
Problems 123

6. Diffusive Mass Transfer **125**

6.1 Remarks on mass transfer 126
6.2 Higher order effects 127

6.3 Conservation of mass equation 128
Examples 129
Problems 131

7. Inverse Heat Transfer 133

7.1 Parameter estimation 133
7.2 Applications to heat transfer 135
7.2.1 Method of sensitivity coefficients 136
7.3 Least squares approach to Example 7.1 139
7.3.1 Analytical solution of the least squares problem 140
7.3.2 Example with heat generation 143
7.4 Linear and nonlinear inverse problems 144
7.5 Nonlinear diffusion equation 146
7.5.1 One-equation model 149
7.5.2 Two-equation model 149
7.6 Inversion of the 1-equation model 150
7.6.1 Construction of the objective function 151
7.6.2 Sensitivity functions 151
7.6.3 Conjugate gradient method 152
7.6.4 Determination of the step sizes 154
7.6.5 Adjoint problem 155
7.6.6 Convergence check 156
7.6.7 Summary of the numerical algorithm 157
7.7 Results and discussion 157
7.7.1 1-equation model 158
7.7.2 2-equation model 166
7.7.3 Closure 167
References 167
Problems 168

8. Microscale Heat Conduction 171

8.1 Hyperbolic heat conduction 173
8.2 Micro-structure of metals 177
8.2.1 Pulsing heat sources 179
8.2.2 Sample solutions 179
8.2.3 Semi-infinite solid 183
8.3 Two-equation model 184
8.3.1 Heat capacity 185
8.3.2 Thermal conductivity 186
8.3.3 Simplified two-equation model 188
8.4 Pulse heating of thin films 190
References 194
Problems 195

Part II RADIATION

9. Introduction to Radiative Transfer 199

9.1 Nomenclature 201
9.2 Three dimensionality 206
Bibliography 207

10. Black Body Radiation 209

10.1 Planck's law 210
10.2 Special forms of Planck's law 212
10.3 Radiation intensity 214
10.4 Total hemispherical emissive power 216
Examples 217
Problems 219

11. Properties of Non-black Surfaces 221

11.1 Emissivity 222
11.2 Absorptivity 224
11.2.1 Integrated absorptivities 225
11.2.2 Diffuse-gray surfaces 228
Examples 228
Problems 234

12. Energy Exchange between Black Surfaces 235

12.1 Enclosure theory 240
12.1.1 Heat transfer in a three surface enclosure 242
12.1.2 Network analysis 244
12.1.3 Spectral calculations for enclosure theory 246
12.2 Calculation of shape factor 246
12.3 First principles calculation of shape factor 255
12.4 Variable wall temperature 262
Problems 263

13. Energy Exchange between Diffuse-Gray Surfaces 265

13.1 Energy exchange calculation 266
13.1.1 Energy exchange in an enclosure with N surfaces 268
13.1.2 Energy exchange in a three surface enclosure 271
13.2 Enclosure with a specular surface 277
Problems 279

14. Treatment of Variable Wall Temperature **281**

14.1 Diffuse-gray enclosure 283
14.1.1 Energy exchange in a two dimensional enclosure 285
14.1.2 Numerical integration 287
14.1.3 Numerical solution of integral equations 289
Examples 291
Problems 294

15. Treatment of Non-Gray Surfaces **297**

15.1 Spectral radiosity formulation 297
15.2 Band approximation 299
15.2.1 Energy exchange between two spectral surfaces 299
15.2.2 Radiosity formulation 302
15.3 Specified heat flux boundaries 303
15.3.1 Heat flux - temperature formulation 304
Examples 304
Problems 305

16. Gas Radiation **307**

16.1 Equivalent beam length 310
16.2 Enclosure theory in the presence of a radiating gas 311
16.3 Spectral calculation of energy exchanges 316
Examples 318
Problems 320

17. Combined Conduction and Radiation **321**

Examples 321
Problems 327

18. Radiative Transfer Equation **329**

18.1 Introduction 329
18.2 Radiative exchange in a participating medium 330
18.2.1 Attenuation by absorption 330
18.2.2 Attenuation by out-scattering 331
18.2.3 Augmentation by emission 332
18.2.4 Augmentation due to in-scattering 334
18.3 The equation of transfer 335
18.4 General solution of RTE 336
18.5 Conservation of thermal energy 339

18.5.1 Radiative heat flux 340
18.5.2 Divergence of radiative heat flux 340
18.5.3 Radiative equilibrium 341
18.6 Boundary conditions for RTE 344
18.6.1 Opaque surfaces 344
18.6.2 Semitransparent and transparent boundaries 345
18.7 Closure 346
Bibliography 346
Problems 347

19. Exact Solutions of the Radiative Transport Equation 349

19.1 Introduction 349
19.2 Gray medium approximation 350
19.3 Plane parallel medium 354
19.4 Non-scattering plane parallel medium 356
19.4.1 Non-scattering plane parallel media with specified temperature field 356
19.4.2 Non-scattering plane parallel medium under radiative equilibrium 361
19.5 Plane parallel isotropic scattering medium 367
19.5.1 Isotropic scattering medium with specified temperature field 368
19.5.2 Radiative equilibrium of isotropic scattering medium 368
19.6 Plane parallel anisotropic medium 369
Examples 370
Bibliography 376
Problems 377
MATLAB code 378

20. Numerical Solution of the Radiative Transport Equation 381

20.1 Introduction 381
20.2 Discrete ordinates method 382
20.2.1 Formulation using discrete ordinates 382
20.2.2 Representation of boundary conditions 384
20.3 Selection of directions for discrete ordinates 384
20.4 Discrete ordinates method for Cartesian coordinates 388
20.4.1 One-dimensional plane parallel medium 389
20.4.2 Two-dimensional configuration 400
20.4.3 Three-dimensional configuration 407
20.5 Closure 407
Bibliography 407
Problems 408
MATLAB codes 409

21. Combined Conduction, Convection and Radiation 419

21.1 Introduction 419
21.2 Combined conduction-radiation heat transfer 420
21.2.1 Conduction-radiation heat transfer in a one dimensional geometry 424
21.2.2 Two-dimensional conduction-radiation heat transfer 431
21.3 Treatment of combined conduction, convection and radiation 435
21.4 Closure 436
Bibliography 436
Problems 437
MATLAB codes 438

Index 449