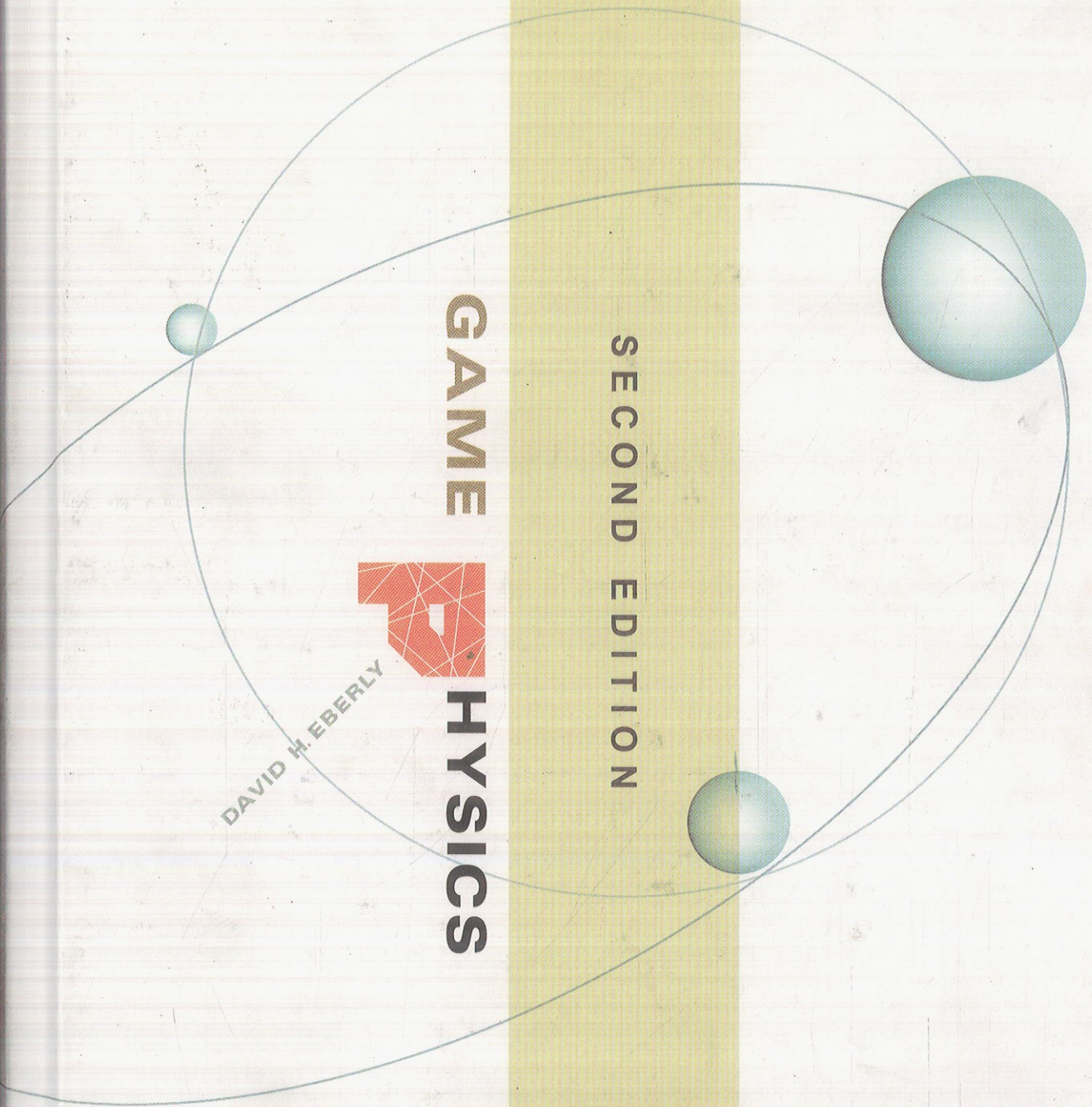


SECOND EDITION

GAME PHYSICS



DAVID H. EBERLY



MK[®]
MORGAN KAUFMANN

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